THE INSTITUTE OF COST AND MANAGEMENT ACCOUNTANTS OF BANGLADESH
CMA JUNE, 2018 EXAMINATION
FOUNDATION LEVEL
SUBJECT: 003. QUANTITATIVE TECHNIQUES

Time: Three hours Full Marks: 100

- Answer any **TEN** questions, FIVE questions from each part.
- Answer must be brief, relevant, neat and clean.
- Use fresh sheet for answering each question.

PART – A: BUSINESS MATHEMATICS

Q. No. 1

(a) Let $\cup = \{1, 2, 3, \ldots\ldots, 19, 20\}$ be the universal set and A = set of all old integers in ∪ and B = set of all prime numbers in ∪. Write down the following sets:
$(i) A \cup B, (ii)\ A \cap B, (iii)\ A^c, (iv)\ (A \cap B)^c$

(b) An investment will yield Tk. 10,000.00 per annum for 8 years. If finance can be obtained at 7% per annum and the investment costs Tk. 50,000.00, is it worth undertaking?

[Marks: (5+5) = 10]

Q. No. 2

(a) Solve $2^x.3^{2x} = 100$

(b) The demand and supply equations are $2\rho^2 + q^2 = 11\ and\ p + 2q = 7$. Find the equilibrium price and quantity. Where p stands for price and q for quantity.

[Marks: (5+5) = 10]

Q. No. 3

(a) Find the value of $\dfrac{1}{\log_{\frac{p}{q}}(x)} + \dfrac{1}{\log_{\frac{q}{r}}(x)} + \dfrac{1}{\log_{\frac{r}{p}}(x)}$

(b) A committee of four is to be chosen from five Science students and three Arts students. In how many ways can this be done so that the committee contains
(*i*) at least one Science students
(*ii*) at least one Science and one Arts students.

[Marks: (4+6) = 10]

Q. No. 4

(a) Find for what values of x, the following expression is maximum and minimum respectively:
$15x^4+8x^3-18x^2$
Find also the maximum and minimum values of the expression.

(b) If tan *A* + sin *A*= *m* and tan *A*- sin *A* = *n*
prove that $m^2-n^2 = 4\sqrt{mn}$

[Marks: (6+4) = 10]

Q. No. 5

(a) In what time will a sum of Tk.1,234.00 amount to Tk.5,678.00 at 8% p.a, compound interest, payable quarterly?

(b) Compute the inverse of Matrix:

$$\begin{pmatrix} 3 & 1 & 3 \\ 2 & 4 & 2 \\ 1 & 2 & 4 \end{pmatrix}$$

[Marks: (4+6) = 10]

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Q. No. 6

(a) If the population of a town increases 25 per thousand per year and the present population is 26,24,000, what will be the population in 3 years' time? What was it a year ago?

(b) Find $\frac{dy}{dx}$, where y =

(*i*) $\frac{1+cos\, x}{\sin x}$; (*ii*) $x^a + a^x + a^a$.

[Marks: 5+(2+3) = 10]

Q. No. 7

(a) (*i*) Find $\frac{dy}{dx}$, $x^2 - y^2 + 3x = 5y$

(*ii*) Evaluate $\int logxdx$.

(b) Marginal revenue function and marginal cost function of a firm are R'(x) = 16 − x^2 and C'(x) = $3x^2 - 2x + 8$ respectively. Where x is the quantity of product and fixed cost of the firm is Tk. 500, then find

(*i*) The total revenue function.
(*ii*) The average revenue function.
(*iii*) The demand function.
(*iv*) The maximum total revenue.

[Marks: (2+2) + 6 = 10]

PART – B: BUSINESS STATISTICS

Q. No. 1

(a) What is data? How can you arrange data?
(b) Distinguish between frequency distribution and relative frequency distribution.
(c) The following table gives the weekly wage in US$ of 50 employees of Beximco Pharmaceuticals Company:

Weekly wage in US$	Number of employees
58.5-63.5	3
63.5-68.5	6
68.5-73.5	5
73.5-78.5	5
78.5-83.5	6
83.5-88.5	9
88.5-93.5	10
93.5-98.5	6
Total	50

Compute the mean, median and mode wage of the employees.

[Marks: (2+2+6) = 10]

Solution No. 1:

(a)

Data:
Data are the raw, disorganized facts and figures collected from any field of inquiry.

The first thing to do with numerical data is to arrange it into a frequency table. Each column of a frequency table generates (is used to create) a particular graph or chart. Data are usually arranged in the form of a frequency table shows the counts (frequencies) of individual categories. Our understanding of the data is further enhanced by calculation of proportion (relative frequency) of observations in each category.

(b)

Frequency Distribution and Relative Frequency Distribution:

- Frequency is the number of times a result occurs, while relative frequency is the number of times the result occurs divided by the number of times the experiment is repeated.
- Frequency can easily be determined by conducting a simple experiment and noting how many times the event is question occurs. On the other hand, relative frequency is determined by using simple division.

(c) The frequency distribution table for calculating mean, median and mode is given below:

Weekly wage in US$	**Number of Employees (f)**	**Mid value (x)**	**f*x**	**Cumulative Frequency**
58.5-63.5	3	61	183	3
63.5-68.5	6	66	396	9
68.5-73.5	5	71	355	14
73.5-78.5	5	76	380	19
78.5-83.5	6	81	486	25
83.5-88.5	9	86	774	34
88.5-93.5	10	91	910	44
93.5-98.5	6	96	576	50
Total	50		4060	

$$\text{Mean of the employees} = \frac{\sum fx}{n} = \frac{4060}{50} = 81.2$$

$$\text{Median of the employees} = 78.5 + \frac{\left(\frac{50}{2}\right)-19}{6} * 5 = 83.5$$

$$\text{Mode of the employees} = 88.5 + \frac{1}{1+4} * 5 = 88.5$$

Q. No. 2

(a) When median are the best measures of frequency distribution?

(b) What is co-efficient of variation?

(c) Find the correct co-efficient of variation from the following information:
Suppose, the mean and standard deviation of a set of 100 observations were worked out as 40 and 5 respectively by a computer which by mistake took the value 50 in place off 40 for one of the observations. Correct standard deviation is 24. You are to find out correct mean and then co-efficient of variation.

[Marks: (2+2+6) = 10]

Solution No. 2:

(a)

When there exists outlier in a frequency distribution median is said to be the best measure of central tendency. Also, median can sometimes be computed even when the data set does not consist of numbers, provided there is natural and appropriate ordering of the data values so that the middle one can be found out. Thus, we can say that median can only be worked out if we have at least ordinal data.

(b)

Co-efficient of Variation (CV):

Co-efficient of variation is the most important measure of relative variation. Developed by Karl Pearson, the coefficient of variation is defined as the ratio of the standard deviation (SD) to the arithmetic mean (AM) usually expressed as percentage and it is denoted by CV. Thus,

$$CV = (SD/AM) * 100$$

We can restate that the CV as the percentage of variation in the AM, the SD being considered as the total variation in the mean.

(c)

Given,

$$\bar{x} = 40$$

$$n = 100$$

Thus,

$$\bar{x} = \frac{\sum x}{n}$$

- $40 = \frac{\sum x}{100}$
- $\sum x = 40*100$

So, $\sum x = 4000$

The correct $\sum x$ = Incorrect $\sum x$ – the wrong value + correct value

= 4000 – 50 + 40

= 3990

Hence, the correct mean is, AM

= 3990/100

= 39.9

And the correct co-efficient of variation is, (SD/AM) * 100

= (24/39.9) * 100
= 60.15.

Q. No. 3

(a) What is business forecasting? What are the assumptions on which business forecasting are made?

(b) "A 12-month moving average of time series data removes trend and cycle." Do you agree? Why or why not?

(c) Catalogues listing textbooks were examined to discover the relationship between the cost of a book and number of pages it contains. The perusal gives the following data for ten books.

Pages	350	270	255	312	190	455	305	210	375	200
Cost(Tk.)	135	110	95	120	70	150	115	80	140	75

(*i*) Obtain the line of regression for estimating the cost of a book.
(*ii*) What is your estimate of cost for a book containing 500 pages?
(*iii*) What increase would you expect in cost if it is decided to increase the number of pages by 100?

[Marks: (2+2+6) = 10]

Solution No. 3:

(a)

Business Forecasting:
Business forecasting refers to the tools and techniques used to predict developments in business, such as sales, expenditures, and profits. The purpose of business forecasting is to develop better strategies based on these informed predictions. Past data is collected and analyzed via quantitative or qualitative models so that patterns can be identified and can direct demand planning, financial operations, future production, and marketing operations.

The assumptions of business forecasting:

- Forecasting is essential to sustainable success
- Business forecast should mirror the business plan
- Quantitative forecasting is concerned with data
- Qualitative business forecasting model is generally used for short-term predictions, or for when data is scarce
- There will always be limitations with forecasting
- One should keep it simple where possible.

(b)
"A 12-month moving average of time series data removes trend and cycle." I do not agree with this statement. Because, moving average is a technique to estimate the trend line. 12-month moving average method deals with time series data considering all four components of a time series. By using this method, one may estimate trend line by considering all components. But, for the purpose of analysis, time series data must be stationary type. To transform stationary data, one may use decomposition or difference methods. In stationary dataset trend, cyclic and seasonal components are absent. This means, moving average technique does not remove trend and cycle from time series data.

(c)

(i) Necessary calculations are given in the following table:

Pages (x)	Cost (Tk.) (y)	x*y	x^2
350	135	47250	122500
270	110	29700	72900
255	95	24225	65025
312	120	37440	97344
190	70	13300	36100
455	150	68250	207025
305	115	35075	93025
210	80	16800	44100
375	140	52500	140625
200	75	15000	40000
$\sum x$ = 2922	$\sum y$ = 1090	$\sum xy$ = 339540	$\sum x^2$ = 918644

Here,

$$\bar{x} = \sum x/n$$
$$= 2922 / 10$$
$$=292.2$$

$$\bar{y} = \sum y/n$$
$$= 1090 / 10$$
$$= 109$$

Thus,

$$\hat{\beta} = \frac{\sum xy - n\bar{x}\bar{y}}{\sum x^2 - n\bar{x}^2}$$
$$= \frac{339540 - 10*292.2*109}{918644 - 10*292.2^2}$$
$$= 0.325$$

And,

$$\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$$
$$= 109 - 0.325*292.2$$
$$= 14.035$$

Hence, the regression line for estimating the cost of a book is,

$$\hat{y} = 14.035 + 0.325\ x$$

(ii) Given,

$$x = 500$$

Then, the estimate of cost is,

$$\hat{y} = 14.035 + 0.325\ x$$
$$= 14.035 + 0.325*500$$
$$=176.535$$
$$\approx 177$$

Thus, the estimate of cost is 177 Tk. (approximately).

(iii) If the number of pages is increased by 100 then one should expect the increase of cost by, $\hat{\beta}$ * 500 = 0.325 * 500 = 162.5 amount.

Q. No. 4

(a) What do you understand by the term "regression analysis"? Point out the role of regression analysis in business decision making.

(b) Find the correlation co-efficient between age and playing habits of the following students.

Ages	12	13	14	15	16	17
No. of students	300	250	200	150	120	100
Regular players	200	150	90	50	40	20

(i) Identify and comment on the nature of correlation.

(ii) Explain with graph and example of (*i*) Zero correlation (*ii*) Perfect correlation and (*iii*) High correlation between above two variables.

[Marks: {2+(5+3)} = 10]

Solution No. 4:

(a)

Regression Analysis:
Regression analysis is a statistical technique for investing and modeling the relationship between variables.

The role of regression analysis in business decision making:
It helps businesses understand the data points they have and use them. Specially the relationship between data points, to make better decisions including anything from predicting sales to understand inventory levels and supply and demand.

(b)

(i) Necessary calculations are given in the following table:

Age (x)	Playing Habits (y) = $\frac{Regular\ players}{No\ of\ students}$	Mean of Age ($\bar{x}$)	Mean of Playing Habits ($\bar{y}$)	x-$\bar{x}$	y-$\bar{y}$	$(x-\bar{x})^2$	$(y-\bar{y})^2$	(x-$\bar{x}$)(y-$\bar{y}$)
12	0.667			-2.5	0.236	6.25	0.056	-0.590
13	0.600			-1.5	0.169	2.25	0.029	-0.254
14	0.450			-0.5	0.019	0.25	0.0001	0.009
15	0.333	14.5	0.431	0.5	-.098	0.25	0.009	-0.049
16	0.333			1.5	-.098	2.25	0.009	-0.049
17	0.200			2.5	-.231	6.25	0.053	-0.579
$\sum x$ = 87	$\sum y$ = 2.583					$\sum(x-\bar{x})^2$ = 17.5	$\sum(y-\bar{y})^2$ = 0.156	$\sum(x-\bar{x})(y-\bar{y})$ = -1.627

Thus, the correlation coefficient,

$$r_{xy} = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sqrt{\sum(x-\bar{x})^2 \sum(y-\bar{y})^2}}$$

$$= \frac{-1.627}{\sqrt{17.5*0.156}} = -0.985$$

Hence, the correlation between age and playing habits is -0.985 which is a strong negative correlation.

(ii)

Zero Correlation:

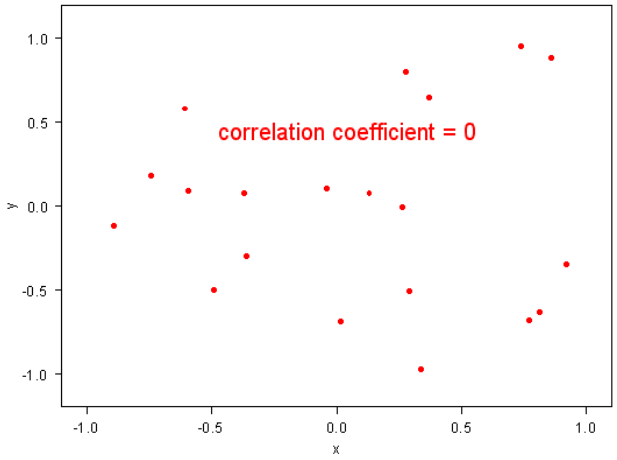

Figure: Zero correlation

This graph shows a perfect zero or no relationship between X and Y and thus the value of correlation is $r_{xy} = 0$.

Perfect positive correlation:

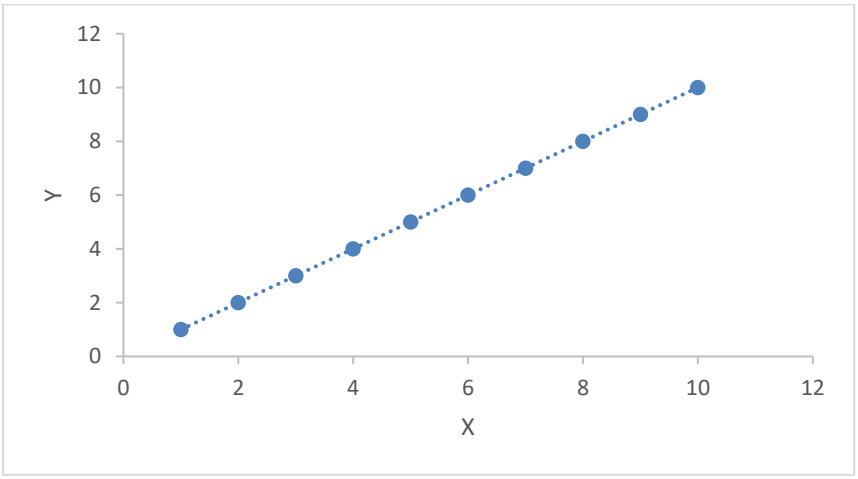

Figure: Perfect positive correlation

This graph shows a perfect positive relationship between X and Y and thus the value of correlation is r_{xy} = +1.

Perfect Negative Correlation:

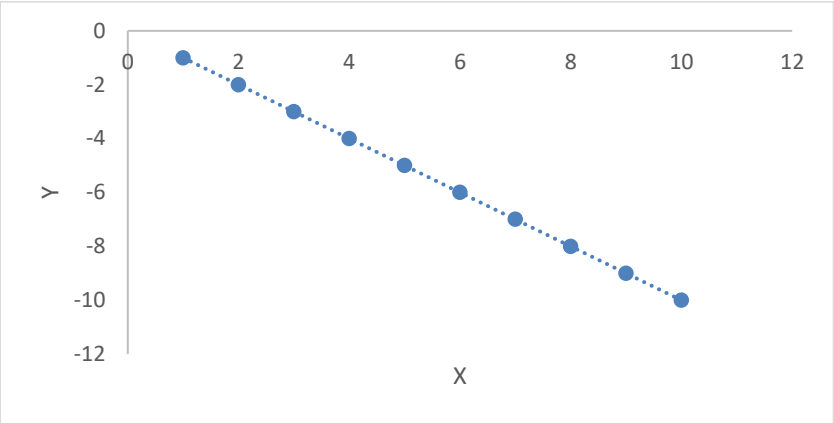

Figure: Perfect negative correlation

This graph shows a perfect negative relationship between X and Y and thus the value of correlation is r_{xy} = -1.

Higher Correlation:

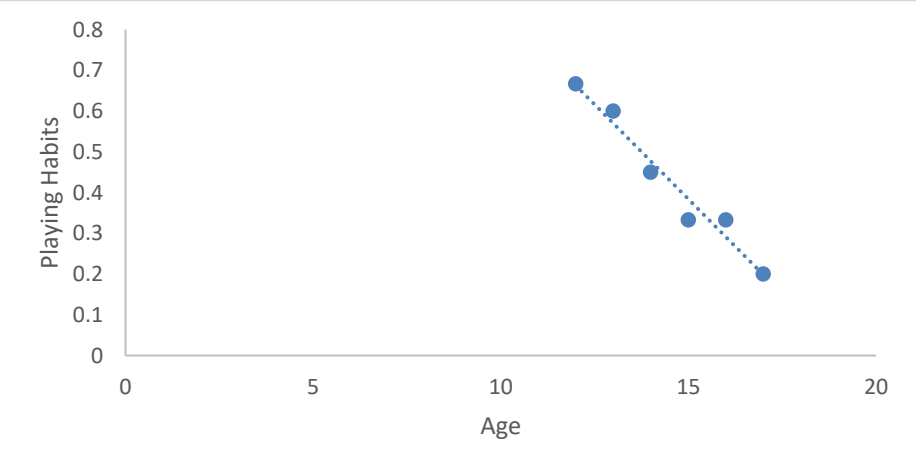

Figure: Correlation between age & playing habits

This graph shows a strong negative relationship between age & playing habits and the value of correlation coefficient is, r_{xy} = -0.985.

Q. No. 5

(a) What is test of hypothesis? Explain type one error and type two error with table.

(b) The nine items of a sample had the following values:

45, 47, 50, 52, 48, 47, 49, 53, 50.

The sample mean is 49 and the sum of squares of deviation taken from mean is 52. Can this sample be regarded as taken from the population having 47 as mean? Also find 95% confidence interval. The table value of 't' for 8 degrees of freedom at 5% level is 2.31.

[Marks: (4+6) = 10]

Solution No. 5:

(a)

Test of Hypothesis:

A statistical hypothesis is an assertion or statement about a population or equivalently about the probability distribution characterizing a population, which we want to verify on the basis of information contained in a sample.

Table

Decision based on sample	Truth about the population		
		Null is true	Alternative is true
	Reject null hypothesis	Type I error	Correct decision
	Accept null hypothesis	Correct decision	Type II error

(b)

Here,

$$\bar{x} = 49$$

$$Var(x) = \frac{52}{8} = 6.5$$

$$SD(x) = 2.55$$

H_0: $\mu = 47$

Vs

H_1: $\mu \neq 47$

Critical region: $t_{tab} > 2.31$

Test statistic

$$t = \frac{\bar{x} - 47}{\frac{SD(X)}{\sqrt{9}}} \sim t_{9-1}$$, Under H_o

$$= \frac{49-47}{\frac{2.55}{3}}$$
$$= 2.353$$

Here, $t > t_{tab}$. Reject H_0.

That is the mean of this population is not 47 at 5% level of significance.

95% confidence interval $(\bar{x} - 2.31 \times SD(\bar{x}), \bar{x} + 2.31 \times SD(\bar{x}))$

$$(49 - 2.31 \times 2.55, 49 + 2.31 \times 2.55)$$
$$= (43.1095, 54.8905)$$

Q. No. 6

(a) Define different types of probabilities.
(b) Explain addition and multiplication law of probabilities.
(c) Write short note on: (*i*) Uniform distribution (*ii*) Uses of moment (*iii*) Normal distribution (*iv*) Lorenz curve (v) Questionnaire and (*vi*) Time series analysis.

[Marks: (2+2+6) = 10]

Solution No. 6:

(a)

There are four types of probabilities. They are:

- **Classical or mathematical probability:**
  If outcomes of S are equally likely (have equal opportunity) then,
  P(A) = $\frac{No.\ of\ outcomes\ in\ A}{No.\ of\ outcomes\ in\ S} = \frac{m_A}{m_S}$
- **Frequency or statistical or empirical probability:**
  When outcome of S is not equally likely then,
  P(A) = $\frac{No.\ of\ times\ A\ occurs}{Total\ No.\ of\ times\ experiment\ is\ repeated} = \frac{n_A}{n}$
- **Subjective probability:**
  In this type of probability personal judgement and degree of belief is used.
- **Axiomatic probability:**
  Let us suppose, S be the sample space of a random experiment and P(A) be the probability of an event A. Then P(A) follows the following three axioms:
  1. $0 \leq P(A) \leq 1$
  2. $P(S) = 1$
  3. If A_1 and A_2 are mutually exclusive then, $P(A_1 \cup A_2) = P(A_1) + P(A_2)$.

(b)

Addition Law of Probability:
The additional laws of probability are:

- $P(A \cup B) = P(A) + P(B)$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Multiplication Law of Probability:
The multiplication laws of probability are:

- For independent events,
  $P(A \cap B) = P(A) * P(B)$

- For dependent events,
  $P(A \cap B) = P(A) * P(B|A)$

(c)

(i) **Uniform Distribution:**

In probability theory and statistics, the continuous uniform distribution is a family of symmetric probability distributions. The distribution describes an experiment where there is an arbitrary outcome that lies between certain bounds. The bounds are defined by the parameters a and b which are the minimum and maximum values. The pdf of uniform distribution is given below:

$$f(x) = \frac{1}{b-a} \text{ ; } a \leq x \leq b$$

Mean and variance of uniform distribution are:

$$E(x) = \frac{a+b}{2} \text{ \&}$$

$$Var(x) = \frac{(b-a)^2}{12} \text{ respectively.}$$

(ii) **Uses of Moments:**

The uses of moment are given below:

- Moments are used in specifying a distribution and describing its properties.
- They are used in many theoretical distributions and in the fitting of a particular distribution of experimental data.
- For lower orders, the moments of classical densities can be expressed simply in terms of the parameters.
- Raw and central moments are used both for discrete and continuous variable.
- Factorial moment is used for discrete variables.

(iii) **Normal Distribution:**

In probability theory a normal or Gaussian distribution is a type of continuous probability distribution for a real valued random variable. The general form its probability density function is,

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} exp\ \{-\frac{1}{2}\ (\frac{x-\mu}{\sigma})^2\}$$

The parameter μ is the mean or expectation of the distribution (and also its median and mode) and the parameter σ is its standard deviation. The variance of the distribution is σ^2.A random variable with a Gaussian distribution is said to be normally distributed and is called a normal deviate.

(iv) **Lorenz Curve:**

A Lorenz curve is a graphical representation of income inequality or wealth inequality developed by American economist Max Lorenz in 1905. The graph plots percentile of the population on the horizontal axis according to income or wealth. It plots cumulative income or wealth in the vertical axis, so that an x value of 45 and y value of 14.2 would mean that the bottom 45% of the population controls 14.2% of the total income or wealth. In practice a Lorenz curve is usually a mathematical function estimated form an incomplete set of observations of income or wealth. A graph of Lorenz curve is given below:

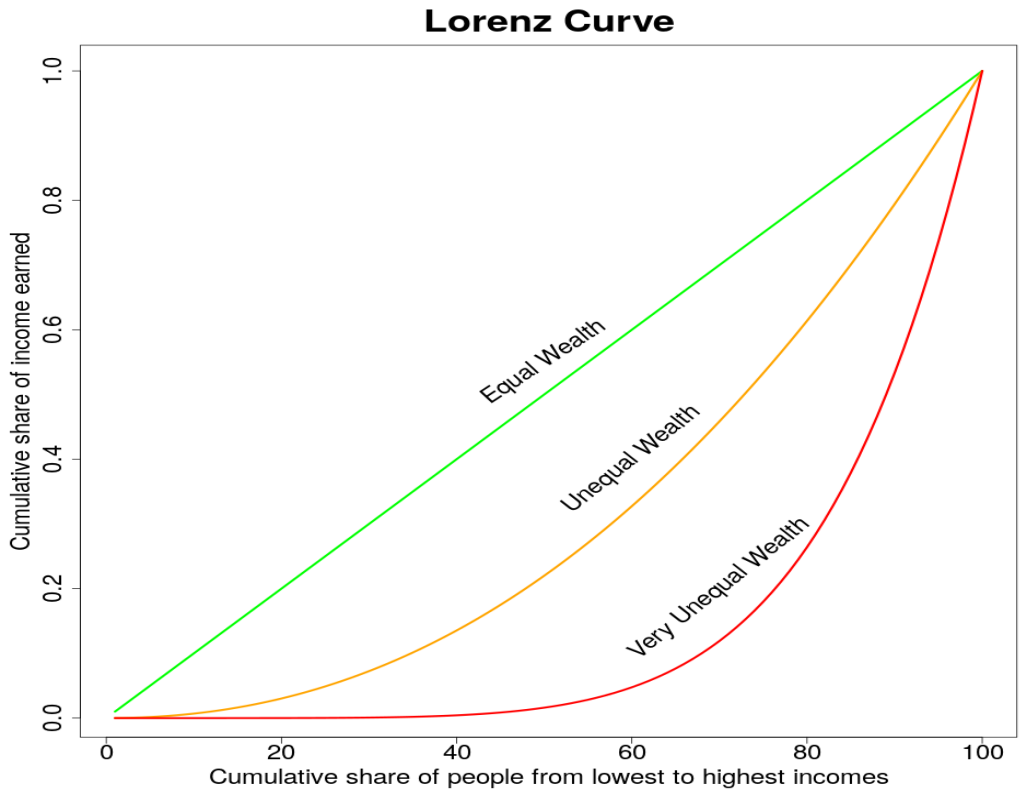

Figure: Lorenz Curve

(v) **Questionnaire:**

A questionnaire is a research instrument consisting of a series of questions (or other types of prompts) for the purpose of gathering information from respondents. The questionnaire was invented by the Statistical Society of London in 1838. Although questionnaires are often designed for statistical analysis of the responses. Questionnaires have advantages over some other types of surveys in that they are cheap, do not require as much effort from the questioner as verbal or telephone surveys, and often have standardized answers that make it simple to compile data.

(vi) **Time Series Data:**

A sequence of observations measured over time (usually at equally spaced intervals, e.g., weekly, monthly and annually). The investigator uses the past data of the target variable to forecast the present and future values of the variable. A time series is an ordered sequence of observations. Although the ordering is usually through time, particularly in terms of some equally spaced time intervals, the ordering may also be taken through other dimensions, such as space.

Q. No. 7

(a) Explain how statistics is useful in the decision making process of business and management.

(b) Distinguish between the f following **(any four):**
(*i*) Sample and Population (*ii*) Point estimation and interval estimation (*iii*) Statistic and parameter (*iv*) Sampling error and non Sampling error (*v*) Histogram and bar diagram (*vi*) Skewness and Kurtosis.

[Marks: (2+8) = 10]

Solution No. 7:

(a)

Statistics are useful when they are applied to improve decision making. No longer in the production of statistics confined to quantitative analysis and market research divisions in firms. Managers in each of the functional areas of business use statistics daily to improve decision making.

(b)

(i) Sample and Population:

Sample	Population
A sample is the specific group that one will collect data from.	A population is the entire group that one wants to draw conclusions about.
The size of sample is always is less than the total size of the population.	The size of the population is greater than the size of the sample.
It is more efficient to collect data from sample.	It is less efficient to collect data from population.
This is a less time-consuming process.	This is a very time-consuming process.
Collecting data from sample is cheaper than collecting data from population.	Collecting data from population is more expensive than collecting data from sample.

(ii) Point Estimation and Interval Estimation:

Point Estimation	Interval Estimation
The point estimation uses a single value.	The interval estimation uses a range of numbers to infer information about the population.
A point estimate of a population parameter is a single value of a statistic.	An interval estimate is defined by two numbers, between which a population parameter is said to lie.
The sample mean x is a point estimate of the population mean μ.	a<x<b is an interval estimate of the population mean μ. It indicates that population mean is less than a and greater than b.

(iii) Statistic and Parameter:

Statistic	Parameter
Statistic is a number that summarize data from a sample.	Parameter is number that summarize data for an entire population.
A statistic is a characteristic of a sample, a portion of the target population.	A parameter is a fixed measure describing the whole population.
Statistic is known number.	Parameter is fixed & unknown numerical value.
In sample statistic, mean is represented by $\bar{x}$.	In population parameter, mean is represented by μ .

(iv) Sampling error and Non sampling error:

Sampling Error	Non-sampling Error
Sampling error is one which occurs due to unrepresentativeness of the sample selected for observation.	Non-sampling error is an error arise from human error, such as error in problem identification, method or procedure used.
This occurs when sample is selected.	This occurs both in sample and census.
This occurs because of deviation between sample mean and population mean.	This occurs because of deficiency and analysis of data.
In this case, possibility of error reduced with the increase in sample size.	It has nothing to do with sample size.

(v) Histogram and Bar Diagram:

Histogram	Bar Diagram
Histogram is used to represent frequency distribution of continuous variable.	Bar diagram is used to represent frequency or number of variable values.
There is no space between two bars or rectangles of a histogram because of continuity of variable values.	The space between two bars or a rectangle of a bar diagram is arbitrary.
In a histogram, the area of bars or rectangles is proportional to its corresponding frequency.	The height of bars in a bar diagram is proportional to its corresponding frequency.

(vi) Skewness and Kurtosis:

Skewness	Kurtosis
Skewness is a measure that refers to the extent of symmetry or asymmetry in a distribution	Kurtosis refers to the pointedness of a peak in the distribution curve.
It is an indicator of lack of equivalence in the frequency distribution.	It is the measure of data, which is either peaked or flat in relation to the normal distribution.
It represents the amount and direction of the skew.	It represents how tall and sharp the central peak is?

= THE END =