ICMAB
THE INSTITUTE OF COST AND MANAGEMENT ACCOUNTANTS OF BANGLADESH

CMA DECEMBER, 2019 EXAMINATION
FOUNDATION LEVEL
SUBJECT: 003. QUANTITATIVE TECHNIQUES

Time: Three hours | Full Marks: 100

- ❖ Answer any **TEN** questions, FIVE questions from each part.
- ❖ Answer must be brief, relevant, neat and clean.
- ❖ Use fresh sheet for answering each question.

PART – A: BUSINESS MATHEMATICS

Q. No. 1

(a) In a recent Unilever survey of 2,000 people, the following information is found:

- 1,200 people like Sunsilk
- 980 like all clear
- 740 like Meril
- 520 like Sunsilk and all clear
- 420 like Sunsilk and Meril
- 340 like all clear and Meril and
- 260 like all three

Find out:

(i) How many people does not like all three?
(ii) How many people like only Meril?

(b) If $a^{2x-1}=b^{1-3y}$ and $a^{3x-1}=b^{2y-2}$, show that $13xy=7x+5y-3$.

[Marks: (5+5) = 10]

Solution:

(a)

Here,

$$Total\ number\ of\ respondents = 2000$$
$$n(S) = 1200$$
$$n(A) = 980$$
$$n(M) = 740$$
$$n(S \cap A) = 520$$
$$n(S \cap M) = 420$$
$$n(A \cap M) = 340$$
$$n(S \cap A \cap M) = 260.$$

i.

Now,

Total number of people like all three Unilever products are

$$n(S \cup A \cup M) = n(S) + n(A) + n(M) - n(S \cap A) - n(S \cap M) - n(A \cap M) + n(S \cap A \cap M)$$
$$= 1200 + 980 + 740 - 520 - 420 - 340 + 260 = 1900$$

Number of people does not like all three= $Total\ number\ of\ respondents - n(S \cup A \cup M)$

$$= 2000 - 1900 = 100$$

ii. Number of people like only Meril

$= n(M) - [n(S \cap M) - n(S \cap A \cap M)] - [n(A \cap M) - n(S \cap A \cap M)] - n(S \cap A \cap M)$

$= n(M) - n(S \cap M) - n(A \cap M) + n(S \cap A \cap M)$

$= 740 - 420 - 340 + 260 = 240$

(b)

Given,

$$a^{2x-1} = b^{1-3y} \cdots\cdots\cdots (1)$$

$$a^{3x-1} = b^{2y-2} \cdots\cdots\cdots (2)$$

From equation (2),

$$a^{x+2x-1} = b^{2y-2}$$

$$\Rightarrow a^x . a^{2x-1} = b^{2y-2}$$

$$\Rightarrow a^x . b^{1-3y} = b^{2y-2} \qquad [Using\ (1)]$$

$$\Rightarrow a^x = \frac{b^{2y-2}}{b^{1-3y}}$$

$$\Rightarrow a^x = b^{2y-2-1+3y}$$

$$\Rightarrow a^x = b^{5y-3}$$

$$\Rightarrow (a^x)^{\frac{1}{x}} = b^{\frac{5y-3}{x}}$$

$$\Rightarrow a = b^{\frac{5y-3}{x}} \cdots\cdots\cdots (3)$$

Putting the value of a in equation (1) one may write,

$$\Rightarrow \left(b^{\frac{5y-3}{x}}\right)^{2x-1} = b^{1-3y}$$

$$\Rightarrow b^{\frac{(5y-3)(2x-1)}{x}} = b^{1-3y}$$

$$\Rightarrow \frac{(5y-3)(2x-1)}{x} = 1 - 3y$$

$$\Rightarrow 10xy - 6x - 5y + 3 = x - 3xy$$

$$\therefore 13xy = 7x + 5y - 3$$

[Showed]

Q. No. 2

(a) If $\log \frac{x+y}{3} = \frac{1}{2}(\log x + \log y)$; then show that $\frac{x}{y} + \frac{y}{x} = 7$

(b) Solve the equation:

$$\sqrt{\frac{x}{x+16}} + \sqrt{\frac{x+16}{x}} = \frac{25}{12}$$

[Marks: (5+5) = 10]

<u>Solution:</u>

(a)

Given,

$$\log\frac{x+y}{3}=\frac{1}{2}(\log x+\log y)$$

$$\Rightarrow \log\frac{x+y}{3}=\frac{1}{2}(\log xy)$$

$$\Rightarrow 2\log\frac{x+y}{3}=\log(xy)$$

$$\Rightarrow \log\left(\frac{x+y}{3}\right)^2=\log(xy)$$

$$\Rightarrow \frac{(x+y)^2}{9}=xy$$

$$\Rightarrow \frac{x^2+2xy+y^2}{xy}=9$$

$$\Rightarrow \frac{x}{y}+2+\frac{y}{x}=9$$

$$\therefore \frac{x}{y}+\frac{y}{x}=7$$

[Showed]

(b)

Here,

$$\sqrt{\frac{x}{x+16}}+\sqrt{\frac{x+16}{x}}=\frac{25}{12}$$

$$Let, \quad \frac{x}{x+16}=a$$

$$\Rightarrow \sqrt{a}+\frac{1}{\sqrt{a}}=\frac{25}{12}$$

$$\Rightarrow \frac{a+1}{\sqrt{a}}=\frac{25}{12}$$

$$\Rightarrow 12a+12=25\sqrt{a}$$

$$\Rightarrow \{12(a+1)\}^2=625a \qquad [Squaring\ both\ sides]$$

$$\Rightarrow 144(a^2+2a+1)=625a$$

$$\Rightarrow 144a^2+288a-625a+144=0$$

$$\Rightarrow 144a^2-337a+144=0$$

$$\Rightarrow 144a^2-256a-81a+144=0$$

$$\Rightarrow 16a(9a-16)-9(9a-16)=0$$

$$\Rightarrow (16a-9)(9a-16)=0$$

$$Either\ a=\frac{9}{16}, or\ a=\frac{16}{9}$$

$$When\ a = \frac{9}{16} \Rightarrow \frac{x}{x+16} = \frac{9}{16}$$

$$\Rightarrow 16x - 9x = 144$$

$$\Rightarrow 7x = 144$$

$$\therefore x = \frac{144}{7}$$

$$When\ a = \frac{16}{9} \Rightarrow \frac{x}{x+16} = \frac{16}{9}$$

$$\Rightarrow 9x = 16x + 256$$

$$\Rightarrow 7x = -256$$

$$\therefore x = \frac{-256}{7}$$

Therefore, $x = \frac{144}{7}, \frac{-256}{7}$

Q. No. 3

(a) If A = $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$, then show that, $A.A^{-1} = I$

(b) Find the equation of the line which passes through the points (3,1) and the intersection of the lines $4y - 3x + 22 = 0$ and $x - y - 6 = 0$.

[Marks: (5+5) = 10]

<u>Solution:</u>

(a)

Here,

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$$

$$\therefore |A| = 3.$$

and,

$$A^{-1} = \begin{bmatrix} \frac{11}{3} & -3 & \frac{1}{3} \\ -\frac{7}{3} & 3 & -\frac{2}{3} \\ \frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix}$$

Now,

$$AA^{-1} = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix} \begin{bmatrix} \frac{11}{3} & -3 & \frac{1}{3} \\ -\frac{7}{3} & 3 & -\frac{2}{3} \\ \frac{2}{3} & -1 & \frac{1}{3} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{11}{3} - \frac{14}{3} + 2 & -3 + 6 - 3 & \frac{1}{3} - \frac{4}{3} + 1 \\ \frac{11}{3} - 7 + \frac{10}{3} & -3 + 9 - 5 & \frac{1}{3} - 2 + \frac{5}{3} \\ \frac{11}{3} - \frac{35}{3} + 8 & -3 + 15 - 12 & \frac{1}{3} - \frac{10}{3} + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

[Showed]

(b)

Given that,

$$4y - 3x + 22 = 0 \ldots\ldots\ldots (1)$$

$$x - y - 6 = 0 \ldots\ldots\ldots (2)$$

From equation (2) one may write,

$$x = y + 6 \ldots\ldots\ldots (3)$$

Using equation (3) in (1),

$$4y - 36y - 18 + 22 = 0$$

$$\Rightarrow y + 4 = 0$$

$$\therefore y = -4$$

Putting $y = -4$ in equation (3) one may get,

$$x = -4 + 6 = 2$$

Thus, the intersecting point of the two lines are $(x, y) = (2, -4)$

Therefore, the equation of the line which passes through the points $(3,1)$ and $(2, -4)$ is

$$\frac{y - 1}{1 + 4} = \frac{x - 3}{3 - 2}$$

$$\Rightarrow \frac{y - 1}{5} = x - 3$$

$$\Rightarrow y - 1 = 5x - 15$$

$$\therefore y - 5x + 14 = 0.$$

Q. No. 4

(a) A committee of 5 members is to be formed out of 5 males and 6 females. Find the number of ways in which it can be done so that among the members chosen in the committee there are:

(i) 3 males and 2 females

(ii) 2 males

(iii) No female

(iv) At least one female

(v) Not more than 3 males

(b) Assume that the interest rate is 6% per annum. Then how much money do you need to invest to make difference between simple interest and compound interest for 2 years be Tk. 13.50. Also find the amount of simple interest and compound interest.

[Marks: (5+5) = 10]

<u>Solution:</u>

(a)

Here,

Number of males = 5, number of females = 6

i) $5C3 * 6C2 = 10 * 15 = 150\ ways$

ii) $5C2 * 6C4 = 10 * 15 = 150\ ways$

iii) $5C5 * 6C0 = 1\ way$

iv) $5C0 * 6C5 + 5C1 * 6C4 + 5C2 * 6C3 + 5C3 * 6C2 + 5C4 * 6C1$

$= 6 + 5 * 15 + 10 * 20 + 10 * 15 + 5 * 6 = 6 + 75 + 200 + 150 + 30 = 461\ ways$

v) $5C0 * 6C5 + 5C1 * 6C4 + 5C2 * 6C3 + 5C3 * 6C2 = 6 + 75 + 200 + 150 = 431\ ways$

(b)

Let,

$$P = Total\ investment;$$

$Given, Time = 2\ years\ and\ interest\ rate, r = 6\% = 0.06$

$I = simple\ interest = P * time * rate = P * 2 * 0.06$

$Compound\ interest = C - P = P(1 + 0.06)^2 - P; \quad where\ C = P(1 + rate)^{time}$

$= P[1.06^2 - 1]$

According to question,

$$compound\ interest - simple\ interest = 13.50$$

$$\Rightarrow P[1.06^2 - 1] - P * 2 * 0.06 = 13.50$$

$$\Rightarrow P[1.1236 - 1 - 0.12] = 13.50$$

$$\Rightarrow P = \frac{13.50}{0.0036} = 3750$$

Total investment = 3750

Simple Interest after 2 years is = $3750 * 2 * 0.06 = 450$

Compound interest after 2 years = $3750 * (1.06)^2 - 3750 = 4213.5 - 3750 = 463.5$

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Q. No. 5

(a) Find $\frac{dy}{dx}$, given that (i) $y = x^3 \sin x$, (ii) $y = \frac{\log x}{\sin x}$.

(b) If $u(x, y) = x^2 - y^2 - 2xy - 2x + 3y$, find $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$.

(c) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\ln x}{(1 + \ln x)^2}$.

[Marks: (1.5+1.5)+4+3 = 10]

<u>Solution:</u>

(a)

i)

Given,

$$y = x^3 \sin x$$

$$\frac{dy}{dx} = x^3 \frac{d}{dx}(\sin x) + \sin x \frac{d}{dx}(x^3)$$

$$= x^3 \cos x + 3x^2 \sin x$$

ii)

Given,

$$y = \frac{\log x}{\sin x}$$

$$\frac{dy}{dx} = \frac{sinx \ \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(\sin x)}{sin^2 x}$$

$$= \frac{sinx \, . \frac{1}{x} + \log x \, . \cos x}{sin^2 x}$$

(b)

Here,

$$u(x, y) = x^2 - y^2 - 2xy - 2x + 3y$$

$$\frac{\partial u}{\partial x} = 2x - 0 - 2y - 2 + 0 = 2x - 2y - 2$$

$$\frac{\partial^2 u}{\partial x^2} = 2$$

$$\frac{\partial u}{\partial y} = 0 - 2y - 2x - 0 + 3 = 3 - 2y - 2x$$

$$\frac{\partial^2 u}{\partial y^2} = -2$$

Therefore,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 2 - 2 = 0$$

(c)

Given,

$$x^y = e^{x-y}$$

$$\ln x^y = \ln e^{x-y} \quad [taking \log on\ both\ sides]$$

$$y \ln x = x - y \quad \left[\Rightarrow y = \frac{x}{1+\ln x}\right]$$

$$\frac{y}{x} + \ln x \frac{dy}{dx} = 1 - \frac{dy}{dx}$$

$$\ln x \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{y}{x}$$

$$\frac{dy}{dx}(1 + \ln x) = 1 - \frac{\frac{x}{1+\ln x}}{x} \qquad \left[Using \quad y = \frac{x}{1+\ln x}\right]$$

$$\frac{dy}{dx}(1 + \ln x) = 1 - \frac{1}{1+\ln x} = \frac{\ln x}{1+\ln x}$$

$$\frac{dy}{dx} = \frac{\ln x}{(1+\ln x)^2}$$

Q. No. 6

(a) Demand for goods of an industry is given by the equation $pq = 100$ where *p* is the price and *q* is the quantity. Supply is given by the equation 20 + 3*p* = *q*. What is the equilibrium price and quantity?

(b) Determine whether the function $y = 3x^2 - 25$ is increasing or decreasing at the point x=3.

(c) If one root of the equation $3x^2 - kx + 4 = 0$ is 3 times the other, find the value of k.

[Marks: (4+3+3) = 10]

<u>Solution:</u>

(a)

Here,

The equation for demand is, $pq = 100 \Rightarrow pq - 100 = 0 \quad , where\ p = price; q = quantity$

and the equation for supply is, $20 + 3p = q \Rightarrow 20 + 3p - q = 0$

In equilibrium,

$$Supply = Demand$$

$$pq - 100 = 20 + 3p - q$$

$$pq - 3p + q - 120 = 0$$

$$p(20 + 3p) - 3p + 20 + 3p - 120 = 0 \; [Using\ supply\ equation]$$

$$3p^2 + 20p - 100 = 0$$

$$3p^2 + 30p - 10p - 100 = 0$$

$$(p + 10)(3p - 10) = 0$$

$$p = \frac{10}{3}\ as\ price\ cannot\ be\ negative$$

$$Equilibrium\ price = \frac{10}{3}$$

Thus,

$$q = 20 + 3 * \frac{10}{3} = 30$$

$$Equilibrium\ supply = 30$$

(b)

Given,

$$f(x) = y = 3x^2 - 25$$

$$\therefore\ f'(x) = \frac{dy}{dx} = 6x$$

At the point $x = 3,\ f'(x) = 18 > 0$

So, the function will be increasing at point $x = 3$.

(c)

Let,

one root of the given equation is a and the other root is $3a$.

Using the relation between roots and coefficients of quadratic equation, we have

$$a + 3a = -\frac{-k}{3} \dots\dots\dots\dots (1)$$

$$a * 3a = \frac{4}{3} \dots\dots\dots\dots (2)$$

Now,

from equation (1)

$$4a = \frac{k}{3}$$

$$k = 12a$$

and from equation (2)

$$a^2 = \frac{4}{9}$$

$$a = \pm\frac{2}{3}$$

Therefore,

$$k = 12\left(\pm\frac{2}{3}\right) = \pm 8$$

Q. No. 7

(a) Total cost of production of x units $\frac{x^3}{3} - 6x^2 - 4x + 100$ and sales revenue (8-4x). How many products are to be produced to earn maximum profit?

(b) Evaluate of the following integrals:

(i) $\int \frac{2}{x(2-x)} dx,$ (ii) $\int \frac{dx}{(1+x^2)\sqrt{\tan^{-1} x + 3}}$;

(c) Mr. Samium borrows Tk. 100,000 from Eastern Bank Limited at 12% compound interest rate which will be repaid over next 4 years. Payment will be made in equal installment at the end of each month. How much he needs to pay in each installment?

[Marks: (3+4+3) = 10]

Solution:

(a)

Here,

$$cost = \frac{x^3}{3} - 6x^2 - 4x + 100$$

$$revenue = 8 - 4x$$

$$Profit, y\ (say) = revenue - cost$$

$$= 8 - 4x - \frac{x^3}{3} + 6x^2 + 4x - 100$$

$$y = 6x^2 - \frac{x^3}{3} - 92$$

For profit maximization,

$$\frac{dy}{dx} = 12x - \frac{3x^2}{3} = 0$$

$$\Rightarrow 12x - x^2 = 0$$

$$\Rightarrow x(12 - x) = 0$$

$$\therefore x = 0 \ or \ x = 12$$

Now,

$$\frac{d^2y}{dx^2} = 12 - 2x$$

$$when\ x = 0, \frac{d^2y}{dx^2} = 12 > 0;$$

$$when\ x = 12, \frac{d^2y}{dx^2} = -12 < 0$$

For the value of $x = 12, \frac{d^2y}{dx^2}$ is negative. Thus, profit will be maximized if 12 units of product was produced.

(b)

i) Given,

$$\int \frac{2}{x(2-x)}dx = \int \frac{2+x-x}{x(2-x)}dx = \int \frac{x}{x(2-x)}dx + \int \frac{2-x}{x(2-x)}dx$$

$$= \int \frac{dx}{2-x} + \int \frac{dx}{x} = \ln|2-x| + \ln|x| + constant$$

ii) Given,

$$\int \frac{dx}{(1+x^2)\sqrt{\tan^{-1}x+3}}$$

Let, $y = \tan^{-1}x + 3 \Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2} + 0 \Rightarrow dy = \frac{dx}{1+x^2}$

Now,

$$\int \frac{dy}{\sqrt{y}} = \int y^{-\frac{1}{2}}dy = y^{-\frac{1}{2}+1} + constant = y^{\frac{1}{2}} + constant$$

$$= \sqrt{y} + constant = \sqrt{\tan^{-1}x+3} + constant$$

(c)

Here,

$$A = 1{,}00{,}000\ Tk.$$

$$r = 12\% = 0.12$$

$$n = 4$$

$$C = A(1+r)^n = 1{,}00{,}000(1+0.12)^4 = 157351.9$$

Number of months $= 4 * 12 = 48$

Mr. Saimum need to pay $\frac{157351.9}{48} = 3578.165$ Tk. per month.

PART-B: BUSINESS STATISTICS

Q. No. 1

(a) Explain how statistics is useful in the decision making process of business and management.

(b) Distinguish between the following:

(i) Sample and Population (ii) Skewness and Kurtosis (iii) Statistic and parameter (iv) Histogram and bar diagram.

[Marks: 2+ (2 x 4) = 10]

Solution No. 1:

(a)

The decision-making process must include collection and analysis of as much data and information as possible in order to arrive at optimal business and management decisions. Computerized analysis of data has made the task simpler. The following are a few examples where statistical methods can help in decision making:

1. Random sampling techniques are used by production managers and the QC department to determine quality grades of materials. Accountants use these same techniques while auditing accounts receivables for their clients.
2. Regression and Correlation analysis may be used by the finance department to correlate a set of financial ratios with other business variables.
3. Marketing departments may apply statistical Test of Significance for their market research about a suitable target market for their new products or services.
4. Forecasting techniques may be used by the top management to estimate sales volume for the next budget year.
5. Standard deviation methods are used by various profit centers within the organization to cut down the inherent risk in a particular business decision.

(b)

Population vs sample

Population	Sample
Collection of all individuals or items based on common characteristics.	A representative part of the population
The population is complete set.	The sample is a subset of the population
The measurable quality is called a parameter.	The measurable quality is called a statistic.

Skewness vs Kurtosis

Skewness	Kurtosis
Skewness is a measure that refers to the extent of symmetry or asymmetry in a distribution.	Kurtosis refers to the pointedness of a peak in the distribution curve.
It is an indicator of lack of equivalence in the frequency distribution.	It is the measure of data, which is either peaked or flat in relation to the normal distribution.
It represents the amount and direction of the skew.	It represents how tall and sharp the central peak is?

Statistic vs Parameter

Statistic	Parameter
A statistic is a characteristic of a sample.	A parameter is a fixed measure describing the whole population.
Statistic is a known number and is usually a function of sample values.	Parameter is usually fixed and unknown.
Sample median is statistic.	Population median is parameter.

Histogram vs bar diagram

Histogram	Bar diagram
Histogram represents the frequency distribution of continuous variables.	Bar diagram is a diagrammatic comparison of discrete variables.
It represents numeric data.	It shows categorical data.
Bars are adjacent in histogram.	Bars are separate in bar diagram.

Q. No. 2

(a) What are the various measures of central tendency? Why are they called measures of central tendency? .

(b) When median is the best measure of frequency distribution?

(c) What is co-efficient of variation?

(d) The prices of a company shares in Dhaka and Chittagong stock markets during the last ten months are recoded below:

Month	Jan.	Feb.	March	April	May	June	July	Aug.	Sep.	Oct.
Dhaka(Tk.)	105	120	115	118	130	127	109	110	104	112
Chittagong(Tk.)	108	117	120	130	100	125	125	120	110	135

(i) Determine the arithmetic mean and standard deviation of the prices of shares of each market.

(ii) In which market are the shares prices stable?

[Marks: (2+1+1+6) = 10]

Solution No. 2:

(a)
The most commonly used measures of central tendency are the arithmetic mean, the mode and the median. Less common measures of central tendency include the midrange, the harmonic mean and the geometric mean. These are called central tendency because they indicate the central or average value of a set of data.

(b)
The median is usually preferred to other measures of central tendency when the data set is skewed. Also, when the data set contains outliers then median is the appropriate measure of central tendency.

(c)
The co-efficient of variation (CV) is the ratio of standard deviation to the arithmetic mean. The higher the CV, the greater the level of dispersion around the mean.

(d)
Arithmetic mean for Dhaka, $\bar{X}_{Dhaka} = \frac{\sum_{i=1}^{10} x_i}{10} = \frac{1150}{10} = 115$ and standard deviation,

$$SD_{Dhaka} = \sqrt{\frac{\sum_{i=1}^{10}(x_i - \bar{X}_{Dhaka})^2}{10-1}} = 8.78$$

Arithmetic mean for Chittagong, $\bar{Y}_{Chittagong} = \frac{\sum_{i=1}^{10} y_i}{10} = \frac{1190}{10} = 119$ and standard deviation,

$$SD_{Chittagong} = \sqrt{\frac{\sum_{i=1}^{10}(y_i - \bar{Y}_{Chittagong})^2}{10-1}} = 10.64$$

Coefficient of variation for Dhaka is $CV_{Dhaka} = \frac{SD_{Dhaka}}{\bar{X}_{Dhaka}} \times 100\% = \frac{8.78}{115} \times 100\% = 7.63$

Coefficient of variation for Chittagong is $CV_{Chittagong} = \frac{SD_{Chittagong}}{\bar{Y}_{Chittagong}} \times 100\% = \frac{10.64}{119} \times 100\% = 8.94$

Dhaka stock market shares stable prices compared to Chittagong stock market as $CV_{Dhaka} < CV_{Chittagong}$.

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Q. No. 3

(a) In a certain town, male and female each form 50% of the population. It is known that 20% of the males and 5% of the females are unemployed. A research student studying the employment situation selects an unemployed person at random. The probability data given in table below-

Unemployment data

	Unemployed	Employed	Total
Males	0.100	0.400	0.500
Females	0.025	0.475	0.500
Total	0.125	0.875	1.000

What is the probability that the person so selected is (a) male (b) female?

(b) Length of service in years (X) and the efficiency grades (Y) of eight individual officers of a company are given below:

Serial No.	1	2	3	4	5	6	7	8
Length of service in years (X)	16	12	18	4	3	10	5	12
Efficiency (Y)	23	22	24	17	19	20	18	21

Calculate the karlpearson's co-efficient of correlation and comment on the result.

[Marks: (5+5) = 10]

Solution No. 3:

(a)

Probability of unemployed person, $P(U) = 0.125$

Probability of unemployed male, $P(M) = 0.100$

Probability of unemployed female, $P(F) = 0.025$

(a) The probability of a randomly selected unemployed person will be male $= \frac{P(M)}{P(U)} = \frac{0.100}{0.125} = 0.8$

(b) The probability of a randomly selected unemployed person will be male $= \frac{P(F)}{P(U)} = \frac{0.025}{0.125} = 0.2$

(b)

Pearson's correlation coefficient is

$$r = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum(x_i - \bar{x})^2}\sqrt{\sum(y_i - \bar{y})^2}}$$

$$= 0.951 \quad [Using\ Calculator]$$

This shows that length of service years and efficiency have a strong positive correlation.

Q. No. 4

(a) A study of 241 authors revealed the following data on the distribution of age:

Age (Years)	Number of Authors
Up to 30	20
Up to 40	73
Up to 50	80
Up to 60	44
Up to 70	22
Up to 80	2

Compute the mean, standard deviation and co-efficient of variation of the distribution.

(b) A Tax firm is interested in comparing the quality of work at two of its regional offices. By randomly selecting samples of tax returns prepared at each office and verifying the sample returns accuracy, the firm will be able to estimate the proportion of erroneous returns prepared at each office. Independent random samples from the two offices provide the following information:

	Sample size	Number of returns with errors
Office I	250	35
Office II	300	27

Conduct a hypothesis test to determine whether the error proportions differ between the two offices. (α = 0.10)

[Marks: (5+5) = 10]

Solution No. 4:

(a)

Table: Frequency distribution of authors

Class Interval	Mid value (x_i)	No. of authors (f_i)	$f_i x_i$	$f_i(x_i - \bar{x})^2$
<30	15	20	300	16108.488
30-40	35	73	2555	5126.381
40-50	45	80	3600	209.952
50-60	55	44	2420	5941.074
60-70	65	22	1430	10283.337
70-80	75	2	150	1999.649
Total		n=241	10455	39668.88

Mean, $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{10455}{241} = 43.38$

Standard deviation, $SD = \sqrt{\frac{\sum f_i(x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{39668.88}{240}} = 12.8564$

Coefficient of variation, $CV = \frac{SD}{\bar{x}} \times 100\% = \frac{12.8564}{43.38} \times 100\% = 29.6367\%$

(b)

$H_0{:}\ P_1 = P_2$

Vs

$H_1{:}\ P_1 \neq P_2$

α = 0.10

Critical region $|z| > 1.64$

Here, $p_1 = \frac{35}{250} = 0.14; \quad and\ p_2 = \frac{27}{300} = 0.09$

$$\bar{p} = \frac{35 + 27}{250 + 300} = 0.113$$

Test statistic,

$$z = \frac{p_1 - p_2}{\sqrt{\bar{p}(1-\bar{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} \sim N(0,1); \quad n_1 = 250, n_2 = 300$$

$$= \frac{0.14 - 0.09}{\sqrt{0.113(1-0.113)\left(\frac{1}{250} + \frac{1}{300}\right)}}$$

$$= 1.8442$$

Since $z > \ \alpha$, we reject null hypothesis and can conclude that the error proportions for two groups differ significantly at 10% level of significance.

Q. No. 5

(a) Write down the difference between Discrete variable and Continuous Variable.

(b) From the following data, draw a Histogram, a frequency polygon and a frequency curve:

Age of Service Limited	Below-5	5-10	10-15	15-20	20-25	25-30	30-35
No. of Workers	5	12	25	48	32	6	1

(c) A machine will function only when all the three components A, B and C will work. The probability of a failing during one year for A is 0.15, that of a B is 0.05 and that of C is 0.10. What is the probability that the equipment will fail before the end of the year?

[Marks: (2+5+3) = 10]

<u>Solution No. 5:</u>

(a)

Discrete variable vs Continuous variable

Discrete	Continuous
A variable which can take certain values.	A variable which can take any value in a particular limit.
It jumps from one value to other value, but it will not consider the intermediate value between two values.	Its value increases in fraction but there is no jump.
Example: Number children in a family.	Example: Amount of rain in a day.

(b)
Histogram, frequency polygon and frequency curve are given in these figures.

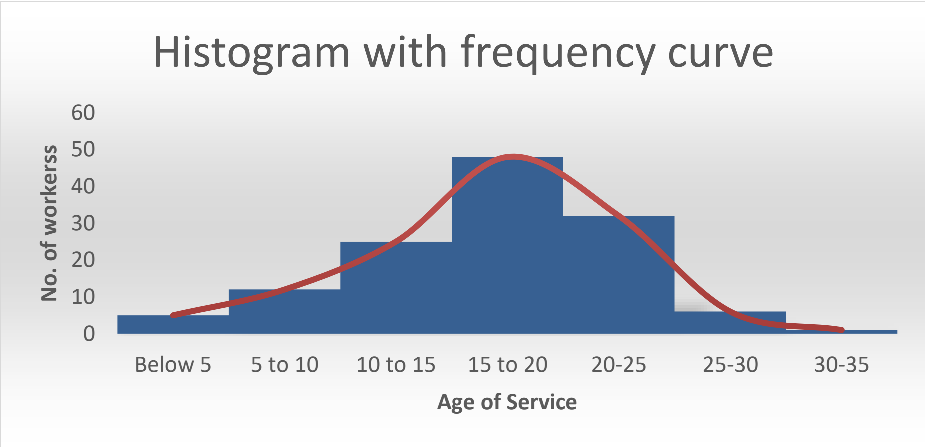

Figure 1: Histogram with frequency polygon

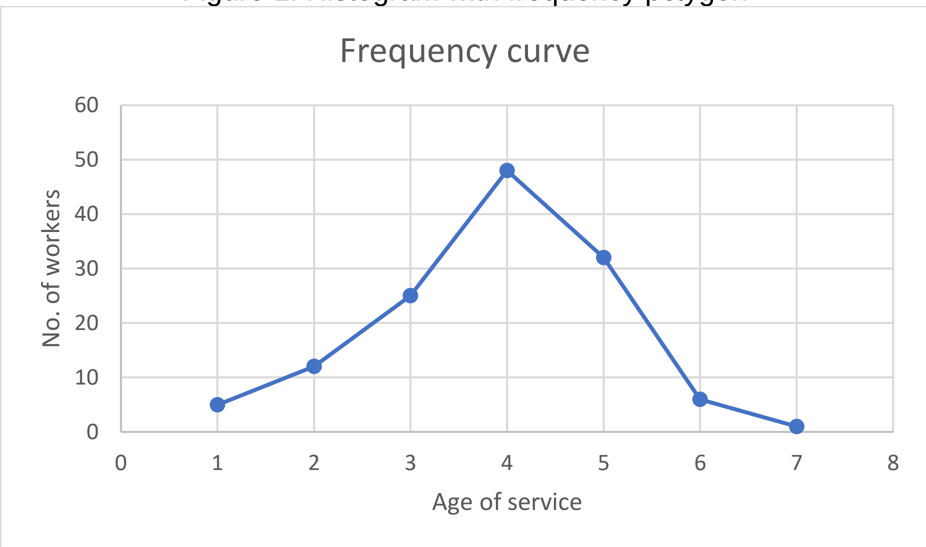

Figure 2: Frequency Curve

(c)
The failing probabilities of each components during one year are $P(A) = 0.15; P(B) = 0.05\ and\ P(C) = 0.10$

Machine will not function if any of the three components will not work.

The that the equipment will fail before the end of the year are

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) = 0.15 + 0.05 + 0.10$$
$$= 0.30$$

Q. No. 6

(a) What is test of hypothesis? Explain type-I error and type-II error with table.

(b) The nine items of a sample had the following values:

45 47 50 52 48 47 49 53 50

The sample mean is 49 and the sum of squares of deviation taken from mean is 52. Can this sample be regarded as taken from the population having 47 as mean? Also find 95% confidence interval. The table value of 't' for 8 degree of freedom at 5% level is 2.31.

[Marks: (3+7) = 10]

<u>Solution No. 6:</u>

(a)

A statistical hypothesis is an assertion or statement about a population or equivalently about the probability distribution characterizing a population, which we want to verify on the basis of information contained in a sample.

Table

Decision based on sample	Truth about the population		
		Null is true	Alternative is true
	Reject null hypothesis	Type I error	Correct decision
	Accept null hypothesis	Correct decision	Type II error

(b)

Here,

$$\bar{x} = 49$$

$$Var(x) = \frac{52}{8} = 6.5$$

$$SD(x) = 2.55$$

H_0: $\mu = 47$

Vs

H_1: $\mu \neq 47$

Critical region: $t_{tab} > 2.31$

Test statistic

$$t = \frac{\bar{x} - 47}{\frac{SD(x)}{\sqrt{9}}} \sim t_{9-1} under\ H_0$$

$$t = \frac{3(49-47)}{2.55} = 2.353$$

Here, $t > t_{tab}$. Reject H_0.

That is the mean of this population is not 47 at 5% level of significance.

95% confidence interval $\left(\bar{x} - 2.31 \times SD(\bar{x}), \bar{x} + 2.31 \times SD(\bar{x})\right)$

$$(49 - 2.31 \times 2.55, 49 + 2.31 \times 2.55) = (43.1095, 54.8905)$$

Q. No. 7

(a) Calculate the skewness from the following data and comment on the result:

Income (Tk.)	200-300	300-400	400-500	500-600	600-700	700-800	800-900
No. of Worker	3	10	25	18	12	7	4

(b) Suppose the head office of Janata Bank Ltd. Collects information of 300 working days from several branch offices operating throughout Bangladesh and the number of frauds found are like that

Number of Fraud	0	1	2	3	4
Number of Days	35	85	120	45	15

Required:

(i) Find the probability distribution of daily fraud occurred?

(ii) What is the probability of not occurring fraud in a day?

(iii) What is the probability of occurring at least one fraud in a day?

[Marks: (4+6) = 10]

<u>Solution No. 7:</u>

(a)

Table: Skewness computation

Income	x_i	f_i	$f_i x_i$	$x_i - \bar{x}$	$f_i(x_i - \bar{x})$	$f_i(x_i - \bar{x})^2$	$f_i(x_i - \bar{x})^3$
200-300	250	3	750	-279.747	-839.241	234774.9	-65677528.6
300-400	350	10	3500	-179.747	-1797.47	323089.2	-58074270
400-500	450	25	11250	-79.7468	-1993.67	158988.9	-12678865.2
500-600	550	18	9900	20.25316	364.557	7383.432	149537.866
600-700	650	12	7800	120.2532	1443.038	173529.9	20867517.6
700-800	750	7	5250	220.2532	1541.772	339580.2	74793612.7
800-900	850	4	3400	320.2532	1281.013	410248.4	131383335
Total		79	41850		0	1647595	90763339.2

$$\bar{x} = 529.7468$$

Third central moment, $\mu_3 = \frac{\sum f_i(x_i - \bar{x})^3}{\sum f_i} = 1148903$

Second central moment, $\mu_2 = \frac{\sum f_i(x_i - \bar{x})^2}{\sum f_i} = 20855.63$

Skewness, $Sk = \frac{\mu_3}{\mu_2^{\frac{3}{2}}} = 0.382$

Since, skewness is greater than zero so the distribution is not symmetric it is positively skewed.

(b)

i) Let, X= number of frauds in a day

Probability distribution

X	0	1	2	3	4
$\Pr[X = x]$	$\frac{35}{300}$	$\frac{85}{300}$	$\frac{120}{300}$	$\frac{45}{300}$	$\frac{15}{300}$

ii)

Probability of not occurring fraud in a day $= \frac{35}{300} = \frac{7}{60}$

ii)

The probability of occurring at least one fraud in a day $= \sum_{x=1}^{4} \Pr[X = x] = \frac{85+120+45+15}{300} = \frac{265}{300} = \frac{53}{60}$

= THE END =