CMA JUNE, 2019 EXAMINATION
FOUNDATION LEVEL
SUBJECT: 003. QUANTITATIVE TECHNIQUES

Time: Three hours Full Marks: 100

- Answer any **TEN** questions, **FIVE** questions from each part.
- Answer must be brief, relevant, neat and clean.
- Use fresh sheet for answering each question.

PART – A: BUSINESS MATHEMATICS (Solution)

Q. No. 1 **[Marks: (6+4) = 10]**

(a) If A = {a, b}, B = {p, q} and C = {q, r}

Find (i) A x (B U C) (ii) (A x B) U (A x C)

(iii) A x (B ∩ C) (iv) (A x B) ∩ (A x C)

Solution:

Given that A = {a, b}, B = {p, q} and C = {q, r}

(*i*) A x (B U C) = {a, b} x ({p, q} U {q, r}) = {a, b} x {p, q, r} = {(a, p), (a, q), (a, r), (b, p), (b, q), (b, r)}

(ii) (A x B) U (A x C)

(A x B) = {a, b} x {p, q} = {(a, p), (a, q), (b, p), (b, q)}

(A x C) = {a, b} x {q, r} = {(a, q), (a, r), (b, q), (b, r)}

Now (A x B) U (A x C) = {(a, p), (a, q), (b, p), (b, q), (a, r), (b, r)}

(iii) A X (B ∩ C)

B ∩ C = {p, q} ∩ {q, r} = {q}

A X (B ∩ C) = {a, b} x {q} = {(a, q), (b, q)}

(iv) (A x B) ∩ (A x C)

(A x B) = {a, b} x {p, q} = {(a, p), (a, q), (b, p), (b, q)}

(A x C) = {a, b} x {q, r} = {(a, q), (a, r), (b, q), (b, r)}

(A x B) ∩ (A x C) = {(a, q), (b, q)}

(b) A Company studies the product preferences of 20,000 consumers. It was found that each of the products A, B, C was liked by 7420, 6230 and 5980 respectively and all the products were liked by 1500. Products A and B were liked by 2580, products A and C were liked by 1200 and products B and C were liked by 1950. Prove that the study results are not correct.

Solution:

Let U = set of consumers who performed in the study

Here, A = set of consumers who liked the product A

B = set of consumers who liked the product B

C = set of consumers who liked the product C

Then A∩B = set of consumers who liked the product A and B

B∩C = set of consumers who liked the product B and C

A∩B∩C = set of consumers who liked the product A, B and C

According to the question

n(U) = 20,000, n(A) = 7420, n(B) = 6230, n(C) = 5980, n(A∩B) = 2580, n(B∩C) = 1950, n(A∩C) = 1200 and n(A∩B∩C) = 1500

Therefore n(A⋃B⋃C) = n(A)+n(B)+n(C)- n(A∩B)- n(B∩C)- n(A∩C)+ n(A∩B∩C)

= 7420+6230+5980-2580-1950-1200+1500 = 21130-5730 = 15,400

Which is contradictory with the number of consumers who performed in the study. Hence the given data is not correct.

Q. No. 2 **[Marks: (5+5) = 10]**

(a) Mr. Shahriar has option of buying a house for Tk. 1 crore on condition that he has to pay Tk. 20 lakh now and the rest of the amount will need to pay with 240 monthly installments, with installments at the end of each month, @ 9% annual compound interest. Find the value of each installment.

Solution:

Here principle amount, A = Tk. 80,00,000, n = 240 months, i = (9/12)% = 0.0075

We know that

$A = R\left[\frac{(1+i)^n-1}{i(1+i)^n}\right] \Rightarrow 80{,}00{,}000 = R\left[\frac{(1+0.0075)^{240}-1}{0.0075(1+0.0075)^{240}}\right] \Rightarrow 80{,}00{,}000 = R\left[\frac{6.01-1}{0.0075\times6.01}\right] \Rightarrow$

$80{,}00{,}000 = R\left(\frac{5.01}{0.045}\right)$

$80{,}00{,}000 = R\left(\frac{5.01}{0.045}\right) \Rightarrow R = \frac{80{,}00{,}000}{111.33} = 718584.44$

Therefore each installment is Tk. 718584.44 (approx.).

(b) Prove that,

$\text{Log}_3 8/(\log_9 16 \log_4 10) = 3\log_{10} 2$

Solution:

Change all logarithms on L.H.S to the base 10 by using the formula $log_b x = \frac{log_a x}{log_a b}$

$$log_3 8 = \frac{log_{10}8}{log_{10}3} = \frac{3log_{10}2}{log_{10}3}$$

$$log_9 16 = \frac{log_{10}16}{log_{10}9} = \frac{4log_{10}2}{2log_{10}3}$$

$$log_4 10 = \frac{log_{10}10}{log_{10}4} = \frac{1}{2log_{10}2}$$

L.H.S = $\frac{log_3 8}{(log_9 16\times log_4 10)} = \frac{3log_{10}2}{log_{10}3} \times \frac{2log_{10}3}{4log_{10}2} \times \frac{2log_{10}2}{1} = 3\ log_{10}2$

Q. No. 3 **[Marks: (5+5) = 10]**

(a) Find the condition that the roots of the equation $ax^2+bx+c = 0$ may differ by 5.

Solution:

Let $ax^2+bx+c = 0$ (1)

Let α and $\alpha + 5$ be the two roots.

Sum of the roots = $2\alpha + 5 = \frac{-b}{a}$ (2)

And the product of the roots = $\alpha^2 + 5\alpha = \frac{c}{a}$ (3)

The condition can be obtained by eliminating α from (2) and (3).

From (2), we have $\alpha = -\frac{b+5a}{2a}$

Substituting in (3) we get $\left(-\frac{b+5a}{2a}\right)^2 + 5\left(-\frac{b+5a}{2a}\right) = \frac{c}{a}; or\ b^2 + 25a^2 + 10ab - 10ab - 50a^2 = 4ac$

$b^2 + 25a^2 + 10ab - 10ab - 50a^2 = 4ac; or b^2 - 25a^2 = 4ac$

(b) Find the inverse of matrix

$$A = \begin{pmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{pmatrix}$$

Solution:

Here $|A| = \begin{vmatrix} 2 & -1 & 3 \\ 4 & 0 & -1 \\ 3 & 3 & 2 \end{vmatrix} = 2(0+3) + 1(8+3) + 3(12-0) = 6 + 11 + 36 = 53.$

The co-factors are

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & -1 \\ 3 & 2 \end{vmatrix} = 0 + 3 = 3;\ A_{12} = (-1)^{1+2}\begin{vmatrix} 4 & -1 \\ 3 & 2 \end{vmatrix} = -(8+3) = -11$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & 0 \\ 3 & 3 \end{vmatrix} = 12 - 0 = 12;\ A_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 3 \\ 3 & 2 \end{vmatrix} = -(-2-9) = 11$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 3 \\ 3 & 2 \end{vmatrix} = 4 - 9 = -5;\ A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -1 \\ 3 & 3 \end{vmatrix} = -(6+3) = -9$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 3 \\ 0 & -1 \end{vmatrix} = 1 - 0 = 1;\ A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 3 \\ 4 & -1 \end{vmatrix} = -(-2-12)$$

$$= 14;\ A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -1 \\ 4 & 0 \end{vmatrix} = (0+4) = 4$$

$$adjA = \begin{pmatrix} 3 & 11 & 1 \\ -11 & -5 & 14 \\ 12 & -9 & 4 \end{pmatrix};\ A^{-1} = \frac{1}{53}\begin{pmatrix} 3 & 11 & 1 \\ -11 & -5 & 14 \\ 12 & -9 & 4 \end{pmatrix}$$

Q. No. 4 **[Marks: (5+5) = 10]**

(a) If x = 3 + 2√2 and y = 1/(3+2√2), find the value of $5x^2+10xy+5y^2$.

Solution:

$$5x^2+10xy+5y^2 = 5(x^2+2xy+y^2) = 5(x+y)^2 = 5\left(3+2\sqrt{2}+1/\left(3+2\sqrt{2}\right)\right)^2$$

$$5x^2+10xy+5y^2 = 5\left(\left(\left(3+2\sqrt{2}\right)^2+1\right)/\left(3+2\sqrt{2}\right)\right)^2 = 5\left((9+12\sqrt{2}+8+1)/3+2\sqrt{2}\right)^2$$

$$5x^2+10xy+5y^2 = 5\left(6\times(3+2\sqrt{2})/\left(3+2\sqrt{2}\right)\right)^2 = 5\times 36 = 180.$$

(b) Mr. Kamal has two investments. He gets 8% interest from XY investment and 10% interest from YZ investment. His total annual return is of Tk. 740. If interest rate changes, i.e. 10% return from XY investment and 8% return from YZ investment, then total return will be Tk. 700. Find the total investment.

Solution:

	XY (x)	YZ (y)	
(i)	8%	10%	740
(ii)	10%	8%	700

Let the number of XY investment is Tk. *x* and YZ investment is Tk. *y*. We can write the two equation form

$$\left.\begin{aligned} x*8\% + y*10\% = 740 \\ x*10\% + y*8\% = 700 \end{aligned}\right\}$$

$$\left.\begin{aligned} x*0.08 + y*0.10 = 740 \\ x*0.10 + y*0.08 = 700 \end{aligned}\right\}$$

Solving the equation we have $x = \text{Tk.}3000$ and $y = \text{Tk.}5000$

Therefore the total investment is $x + y = 3000 + 5000 = \text{Tk.}8000$

Q. No. 5 **[Marks: (5+5) = 10]**

(a) A firm invested Tk. 20,000 in a new factory that has a net return of Tk. 2,000 per year. An investment of Tk. 40,000 would yield a net income of Tk. 8,000 per year. What is the linear relationship between investment and annual income? What would be the return on an investment of Tk. 30,000?

Solution:

Here$P_1 = 20{,}000, I_1 = 2{,}000, P_2 = 40{,}000,\ I_2 = 8{,}000, P_3 = 30{,}000$ and all cases $n = 1$

Now $I_1 = P_1 * n * r_1 \Rightarrow 2000 = 20000 * 1 * r_1 \Rightarrow r_1 = 10\%$

$I_2 = P_2 * n * r_2 \Rightarrow 8{,}000 = 40{,}000 * 1 * r_2 \Rightarrow r_2 = 20\%$

We can write $r_2 = 2r_1$and $P_2 = 2P_1$ and dividing both equation we get

$$\frac{r_2}{P_2} = \frac{r_1}{P_1}$$

Similarly we can write

$\frac{r_1}{P_1} = \frac{r_2}{P_2} = \frac{r_3}{P_3}$

Now take 1st and 3rd equation, we have

$\frac{r_1}{P_1} = \frac{r_3}{P_3}$

$\Rightarrow \frac{10\%}{20000} = \frac{r_3}{30000}$

$$r_3 = 15\%$$

Now the return on investment Tk. 30000 is

$I_3 = P_3 * n * r_3$

$\Rightarrow I_3 = 30000 * 1 * 15\% = \text{Tk.} 4500$

(b) Show that, $(xy)^{\ln x - \ln y} \times (yz)^{\ln y - \ln z} \times (zx)^{\ln z - \ln x} = 1$

Solution:

L.H.S = $(xy)^{\ln x - \ln y} \times (yz)^{\ln y - \ln z} \times (zx)^{\ln z - \ln x}$

$=(z)^{\ln y - \ln x} \times (x)^{\ln z - \ln y} \times (y)^{\ln x - \ln z} \times (xyz)^0$

$= (x)^{\ln z - \ln y} \times (y)^{\ln x - \ln z} \times (z)^{\ln y - \ln x} = 1$

where, $u = (x)^{\ln z - \ln y} \times (y)^{\ln x - \ln z} \times (z)^{\ln y - \ln x}$

$$\ln u = (\ln z - \ln y)\ln x + (\ln x - \ln z)\ln y + \ln z(\ln y - \ln x) = 0 \Rightarrow u = e^0 = 1$$

Q. No. 6 **[Marks: (5+5) = 10]**

(a) A cricket team of 11 players is to be formed from 16 players including 4 bowlers and 2 wicket-keepers. In how many different ways, we can form a team so that the team includes-

(i) Exactly 3 bowlers and 1 wicket-keeper

(ii) At least 3 bowlers and at least 1 wicket-keeper.

Solution:

There are 4 bowlers, 2 wicket-keepers and 10 non-bowlers-non-wicket-keeper

(i) Exactly 3 bowlers from 4 bowlers and 1 wicket-keeper from 2 wicket-keeper and 7 non-bowler-non-wicket-keeper from 10 non-bowler-non-wicket-keeper,

Then there are ${}^4C_3 \times {}^2C_1 \times {}^{10}C_7$ = 960 different ways to form a team.

(ii) Case-1: 3 bowlers from 4 bowlers and 1 wicket-keeper from 2 wicket-keeper and 7 non-bowler-non-wicket-keeper from 10 non-bowler-non-wicket-keeper,

Then there are ${}^4C_3 \times {}^2C_1 \times {}^{10}C_7$ = 960 different ways to form a team.

Case-2: 4 bowlers from 4 bowlers and 1 wicket-keeper from 2 wicket-keeper and 6 non-bowler-non-wicket-keeper from 10 non-bowler-non-wicket-keeper,

Then there are ${}^4C_4 \times {}^2C_1 \times {}^{10}C_6$ = 420 different ways to form a team.

Case-3: 3 bowlers from 4 bowlers and 2 wicket-keeper from 2 wicket-keeper and 6 non-bowler-non-wicket-keeper from 10 non-bowler-non-wicket-keeper,

Then there are ${}^4C_3 \times {}^2C2 \times {}^{10}C_6$ = 840 different ways to form a team.

Case-4: 4 bowlers from 4 bowlers and 2 wicket-keeper from 2 wicket-keeper and 5 non-bowler-non-wicket-keeper from 10 non-bowler-non-wicket-keeper,

Then there are ${}^4C_4 \times {}^2C_2 \times {}^{10}C_5$ = 252 different ways to form a team.

Hence there are total 960+420+840+252 = 2472 different ways to form a team.

(b) If $y = (\cos^{-1} x)^2$, then prove that, $(1 - x^2)y_2 - xy_1$=2.

Solution:

Given that, $y = (\cos^{-1} x)^2$

$$or, y_1 = 2\cos^{-1} x \frac{1}{\sqrt{1 - x^2}}$$
$$or, (1 - x^2){y_1}^2 = (2\cos^{-1}x)^2$$
$$or, (1 - x^2){y_1}^2 = 4y$$
$$or, (1 - x^2)2y_1y_2 - 2x{y_1}^2 = 4y_1$$
$(1 - x^2)y_2 - xy_1$=2

Q. No. 7 **[Marks: (5+5) = 10]**

(a) Beximco Manufacturing Company has examined its cost and revenue structures and determined that total cost C, total revenue R and the number of units produced x are related as follows: $C = 100 + 0.015x^2$ and R = 3 x. Find the level of production that will maximize profit of the company. Also find the profit when x = 120.

Solution:

The given cost and revenue functions are $C = 100 + 0.015x^2$ and R = 3 x

Therefore the profit function is $P(x) = 3x - 0.015x^2 - 100$

$$\frac{dP}{dx} = 3 - 0.03x$$

For critical point $\frac{dP}{dx} = 0; or\ 3 - 0.03x = 0; or\ x = 100$

$\frac{d^2P}{dx^2} = -0.03 < 0$ for all values of x = 100.

Production x = 100 will maximize profit of the company.

Profit for x = 120 is $P(120) = 3 \times 120 - 0.015(120)^2 - 100 = 360 - 316 = 44$

(b) Evaluate:

a. $\int_2^4 \frac{x}{x^2+5} dx$

b. $\int_0^5 (e^x + e^{-x}) dx$

Solution:

(a) $\int_2^4 \frac{x}{x^2+5} dx$

Let z =$x^2 + 5$; or, dz = 2x dx

x	2	4
z	9	21

Now, $\int_2^4 \frac{x}{x^2+5} dx = \int_9^{21} \frac{dz}{2z} = \frac{1}{2}[lnz]_9^{21} = \frac{1}{2}\ (ln21 - ln9)$.

(b) $\int_0^5 (e^x + e^{-x}) dx = e^x - e^{-x}|_0^5 = (e^5 - e^{-5}) - (e^0 - e^{-0}) = e^5 - e^{-5}$

= THE END =