CMA June 2016

PART - A: BUSINESS MATHEMATICS

Q. No. 1

(a) Assume a customer has Tk.5,000 to spend on two brand of cosmetics type A and type B. Each unit of type A costs Tk.450 and each unit of type B costs Tk.750 respectively. Draw his/her budget line. Show what happens to the budget line if the price of type A is cut in half and the price of type B is doubled.

(b) $3^{x+y} = 243,\ 2^{2x-5y} = 8$

Solution: (a) According to the question, we have $450A + 750B = 5000$ (1)

$225A + 1500B = 5000$ (2)

(b) Given $3^{x+y} = 243,\ 2^{2x-5y} = 8$

We can write $3^{x+y} = 3^5$ and $2^{2x-5y} = 2^3$

Therefore, using formula, we get $x + y = 5$ (1) and $2x - 5y = 3$ (2)

Multiplying the first equation by 2, we get $2x + 2y = 10$ (3)

Now subtracting (3) from (2), we get $2x + 2y - (2x - 5y) = 10 - 3 \Rightarrow 7y = 7 \Rightarrow y = 1$

Then from (1), we get $x = 5 - 1 = 4$

Hence, the solution is $(x, y) = (4,1)$.

Q. No. 2

(a) A survey of 600 workers in a plant indicated that 410 owned their own houses, 500 owned cars, 550 owned televisions, 410 owned cars and televisions, 340 owned cars and houses, 370 owned houses and television and 300 owned all three. Illustrate by a Venn diagram and prove that the above data is not correct. What set is empty?

(b) Show that $\dfrac{\tan A}{\sec A - 1} + \dfrac{\tan A}{\sec A + 1} = 2\operatorname{cosec} A$.

Solution: (a) Let H represents a set of workers who owned houses, C represents a set of workers who owned cars and let T represents a set of workers who owned televisions.

Then according to the question, $n(H) = 410$, $n(C) = 500$ and $n(T) = 550$;

$n(C \cap T) = 410$, $n(C \cap H) = 340$, $n(H \cap T) = 300$ and $n(H \cap C \cap T) = 300$.

Using Formula, we get

$$n(H \cup C \cup T) = n(H) + n(C) + n(T) - n(C \cap T) - n(C \cap H) - n(H \cap T) + n(H \cap C \cap T)$$

$$\Rightarrow 600 = 410 + 500 + 550 - 410 - 340 - 370 + 300$$

$\Rightarrow 600 = 1350 - 710 \Rightarrow 600 = 640$ which is absurd. So the given data is not correct.

Results are shown in the Venn diagram where the empty set is shaded.

(b) Left hand side $= \frac{\tan A}{\sec A - 1} + \frac{\tan A}{\sec A + 1} = \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A} - 1} + \frac{\frac{\sin A}{\cos A}}{\frac{1}{\cos A} - 1}$

$$= \frac{\sin A}{1 - \cos A} + \frac{\sin A}{1 + \cos A} = \frac{\sin A(1 + \cos A) + \sin A(1 - \cos A)}{1 - \cos^2 A}$$

$$= \frac{2\sin A}{\sin^2 A} = \frac{2}{\sin A} = 2\cos ecA = \text{Right hand side (Proved).}$$

Q. No. 3

(a) What is the present value of Tk. 2500 payable 4 years from now at 10% interest compounded quarterly?

(b) Demand and supply equations are (q + 20)(p +10) = 400 and q = 2p – 7 respectively, where p stands for price and q for quantity. Find the equilibrium price and quantity.

Solution: (a) Given $A = 2500,\ i = \frac{0.10}{4} = 0.025,\ n = 4 \times 4 = 16$

We have $A = P(1 + \mathrm{i})^{\mathrm{n}} \Rightarrow 2500 = P\left(1 + \frac{0.10}{4}\right)^{16}$

$$\Rightarrow 2500 = P\left(1 + \frac{0.10}{4}\right)^{16} \Rightarrow P = 1684$$

Hence, the present value is Tk. 1,684.

(b) We have, demand function $(q + 20)(p + 10) = 400$ (1)

and supply function is given by $q = 2p - 7$ (2)

From (1), we get

$$(q + 20)(p + 10) = 400 \Rightarrow q + 20 = \frac{400}{p + 10} \Rightarrow q = \frac{400}{p + 10} - 20 = \frac{400 - 20p - 200}{p + 10} = \frac{200 - 20p}{p + 10}$$

Equilibrium conditions are determined by equating demand and supply equations.

Therefore, $\frac{200 - 20p}{p + 10} = 2p - 7 \Rightarrow 2p^2 + 13p - 70 = 200 - 20p \Rightarrow 2p^2 + 33p - 270 = 0$

$$\Rightarrow 2p^2 + 33p - 270 = 0$$

$$\Rightarrow 2p^2 + 45p - 12p - 270 = 0 \Rightarrow 2p(p - 6) + 45(p - 6) = 0 \Rightarrow (p - 6)(2p + 45) = 0 \Rightarrow p = 6$$

and so $q = 12 - 7 = 5$

Hence the equilibrium price and quantity are 6 and 5, respectively.

Q. No. 4

(a) Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 4 \\ 1 & 4 & 3 \end{bmatrix}$$

Solution: (a) First compute $|A| = 1(9-16) - 1(6-12) + 1(8-9) = -2 \neq 0$ so that A^{-1} exists.

The cofactors of A are

$A_{11} = -7,\ A_{12} = 1,\ A_{13} = 1;\ A_{21} = 6,\ A_{22} = 0,\ A_{23} = -2;\ A_{31} = -1,\ A_{32} = -1,\ A_{33} = 1$

Therefore, adjoint matrix, $adj\ A = \begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$

Hence, $A^{-1} = \dfrac{1}{|A|} adj\ A = \dfrac{1}{-2}\begin{bmatrix} -7 & 6 & -1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix} = \begin{bmatrix} 7/2 & -3 & 1/2 \\ -1/2 & 0 & 1/2 \\ -1/2 & 1 & -1/2 \end{bmatrix}$.

(b) A man invested in a share Tk.2000 that has a net return of Tk. 700. An investment of Tk. 5000 would yield a net return of Tk. 1100 for him. What is the linear relationship between investment and net return? What would be the net return on an investment of Tk. 1250?

Solution: Similar problems have been solved.

Q. No. 5

(a) Maximize profits p for a firm, given total revenue $R = 4000q - 33q^2$ and total cost $C = 2q^3 - 3q^2 + 400q + 5000$.

(b) Given the demand function Pd= 25- q^2 and the supply function Ps= 2q+1. Assuming pure competition, find (i) consumer's surplus and (ii) the producer's surplus.

Solution: (a) Given total revenue $R = 4000q - 33q^2$ and total cost $C = 2q^3 - 3q^2 + 400q + 5000$.

Therefore, profit function,

$$p = R - C = 4000q - 33q^2 - 2q^3 + 3q^2 - 400q - 5000 = 3600q - 30q^2 - 2q^3 - 5000$$

To maximize p, calculate $\dfrac{dp}{dq} = 3600 - 60q - 6q^2 = 0 \Rightarrow q^2 + 10q - 600 = 0 \Rightarrow q = 20$

Hence, the maximum profit is $p(20) = 3600 \times 20 - 30(20)^2 - 2(20)^3 - 5000 = 39000$

(b) Given demand function $P_d = 25 - q^2$ and supply function $P_s = 2q + 1$.

Under pure competition, market equilibrium conditions can be obtained by equating the demand and supply function. Therefore,

$$P_d = P_s \Rightarrow 25 - q^2 = 2q + 1 \Rightarrow q^2 + 2q - 24 = 0 \Rightarrow (q+6)(q-4) = 0 \Rightarrow q = 4 = q_0$$

So $p_0 = 25 - (q_0)^2 = 25 - 16 = 9$ and $p_0 q_0 = (9)(4) = 36$

(i) Consumer's Surplus, $CS = \int_0^{q_0} P_d\, dq - p_0 q_0 = \int_0^4 (25 - q^2) dq - 36 = \left[25q - \dfrac{q^3}{3} \right]_0^4 - 36$

$$= 25\times 4 - \frac{1}{3}\times 4^3 - 36 = 64 - \frac{64}{3} = \frac{192-64}{3} = \frac{128}{3}$$

(ii) Producer's Surplus, $PS = p_0 q_0 - \int_0^{q_0} P_s\, dq = 36 - \int_0^4 (2q+1)dq = 36 - \left[q^2 + q\right]_0^4$

$$= 36 - (4^2 + 4) = 36 - 20 = 16$$

Q. No. 6.

(a) Marginal cost is given by MC = 25+30q- 9q^2. Fixed cost is Tk. 55. Find the (i) total cost (ii) average cost and (iii) variable cost functions.

(b) A man borrows Tk. 750 from a money lender and the bill is renewed after every half year at an increase of 21%. What time will elapse before it reaches Tk. 7,500?

Solution: (a) Given marginal cost, $MC = 25 + 30q - 9q^2$ and fixed cost, $FC = 55$.

(i) To find the total cost, we integrate MC with respect to x:

$$TC = \int (25 + 30q - 9q^2)dq = 25q + 30\frac{q^2}{2} - 9\frac{q^3}{3} + k,$$

where k is the fixed cost FC.

Hence, the total cost, $TC = 25q + 15q^2 - 3q^3 + 55$.

(ii) Average cost, $AC = \dfrac{TC}{q} = 25 + 15q - 3q^2 + \dfrac{55}{q}$.

(iii) We know that Total Cost = Variable Cost (VC) + Fixed Cost

$$\Rightarrow 25q + 15q^2 - 3q^3 + 55 = VC + 55 \Rightarrow VC = 25q + 15q^2 - 3q^3.$$

(b) Similar problems have been solved.

Q. No. 7.

(a) Carryout the following: (i) $\int \left(\dfrac{dx}{4x + x^2}\right)$ (ii) $\int \left(x^3 \tan x^4\right) dx$

(b) Evaluate $\dfrac{1}{\log_4 24} + \dfrac{1}{\log_8 24} + \dfrac{1}{\log_{12} 24}$

Solution:

(a) (i) $\displaystyle\int \frac{dx}{4x + x^2} = \frac{1}{4}\int\left(\frac{1}{x} - \frac{1}{x+4}\right)dx = \frac{1}{4}[\ln x - \ln(x+4)] + c = \frac{1}{4}\ln\frac{x}{x+4} + c.$

(ii) To evaluate the given integral, let $x^4 = p$. Then $4x^3 dx = dp \Rightarrow x^3 dx = \frac{1}{4}dp$. Therefore, we get

$$\int \left(x^3 \tan x^4\right) dx = \int \tan p\, dp = \ln(\sec p) + c = \ln(\sec x^4) + c.$$

(b) $\dfrac{1}{\log_4 24} + \dfrac{1}{\log_8 24} + \dfrac{1}{\log_{12} 24} = \dfrac{\log 4}{\log 24} + \dfrac{\log 8}{\log 24} + \dfrac{\log 12}{\log 24} = \dfrac{\log 4 + \log 8 + \log 12}{\log 24}$

$$= \frac{\log 4 + \log 8 + \log 12}{\log 24} = \frac{\log(4\times 8\times 12)}{\log 24} = \frac{\log 24 + \log 16}{\log 24}.$$

CMA DECEMBER, 2016 EXAMINATION

Q. No. 1

(a) Mr. Jahir wants to purchase a machine after 10 years when it will cost Tk. 10,00,000.00. From now, he wants to save money for the machine and plans to deposit money into a bank in 10 equal installments, the first deposit to be made immediately. Calculate the amount of each installment reckoning compound interest at 10% p.a.

(b) Let A = $\begin{pmatrix} 3 & 2 \\ -1 & 0 \end{pmatrix}$

Show that $A^2 - 3A + 2I = O$, when I is the unit matrix of order 2x2 and O is the null matrix of order 2x2.

Solution: (a) Similar problems have been solved.

(b) Calculate $A^2 = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix}$

Therefore, $A^2 - 3A + 2I = \begin{bmatrix} 7 & 6 \\ -3 & -2 \end{bmatrix} - 3\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7-9+2 & 6-6+0 \\ -3+3+0 & -2+0+2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$

Q. No. 2

(a) A person borrows Tk. 20,00,000 on the understanding that he will pay it back in 72 equal monthly installments at the end of each month. If money is worth 14% per annum compounded monthly, find the value on an installment.

(b) Find $\frac{dy}{dx}$ for $e^{x+y} - y^2 \log x^3 = 15$.

Solution: (a) Here $P = 20,00,000$, $i = \frac{0.14}{12} = 0.012$, $n = 72 \times 12 = 864$

We have $P = A\left[\frac{1-(1+\mathrm{i})^{-\mathrm{n}}}{i}\right] \Rightarrow 20,00,000 = A\left[\frac{1-(1.012)^{-864}}{0.012}\right] \Rightarrow A = 24,000$

(b) Given $e^{x+y} - y^2 \log x^3 = 15 \Rightarrow e^{x+y}\frac{d}{dx}(x+y) - [y^2 \frac{d}{dx}\log x^3 + \log x^3 \frac{d}{dx} y^2] = 0$

$$\Rightarrow e^{x+y}\left(1+\frac{dy}{dx}\right) - \left(y^2 \times \frac{3x^2}{x^3} + \log x^3 \times 2y\frac{dy}{dx}\right) = 0$$

$$\Rightarrow e^{x+y}\left(1+\frac{dy}{dx}\right) - \left(\frac{3y^2}{x} + 2y\log x^3 \frac{dy}{dx}\right) = 0$$

$$\Rightarrow \left(e^{x+y} - 2y\log x^3\right)\frac{dy}{dx} = 3\frac{y^2}{x} - e^{x+y} \Rightarrow \frac{dy}{dx} = \frac{3\frac{y^2}{x} - e^{x+y}}{e^{x+y} - 2y\log x^3}.$$

Q. No. 3

(a) A question paper contains 5 questions each having an alternative. In how many ways can an examinee answer one or more questions?

(b) A company studies the product preferences of 25,000 consumers. It was found that each of the products A, B and C was liked by 8,000, 7,000 and 6,000, respectively and all the products were liked by 1500. Products A and B were liked by 2000 and products B and C were liked by 2200. Prove that the study results are not correct.

Solution: (a) Each question have 3 choices (answer the question or answer its alternative or leave the question). So for 5 questions, we have $3\times3\times3\times3\times3=3^5$ ways in total of answering the question.
But in these 3^5 ways, there is one way in which no question is attended.
Hence, the number of ways that a student can answer one or more questions = 3^5-1=242.

(b) Similar problems have been solved.

Q. No. 4

(a) (i) If $y=e^{\tan^{-1}x}$, then show that $(1+x^2)y_2+(2x-1)y_1=0$.

(ii) Find third derivative of the function y = 3x^3 – 5x^2 + 2.

(b) A firm sells all of its product at the rate of Tk. 4 each. The cost function (C) for x units of production is C = 50+1.3x+0.001x^2.

(i) Determine its profit function.
(ii) Determine the unit of production for maximum profit.
(iii) Find its total profit.

Solution: (a) (i) Given $y=e^{\tan^{-1}x}$. Then $\frac{dy}{dx}=e^{\tan^{-1}x}\times\frac{d}{dx}(\tan^{-1}x)=e^{\tan^{-1}x}\times\frac{1}{1+x^2}$

$\Rightarrow(1+x^2)y_1=y\Rightarrow(1+x^2)y_2+2xy_1=y_1\Rightarrow(1+x^2)y_2+(2x-1)y_1=0$.

(ii) Given $y=3x^3-5x^2+2$. Then first derivative, $y_1=9x^2-10x$; second derivative, $y_2=18x-10$ and finally, third derivative, $y_3=18$.

(b) (i) Let x unit be produced. Therefore, to produce x unit, profit, $P=4x-(50+1.3x+0.001x^2)$

(ii) So $\frac{dP}{dx}=4-1.3-0.002x=2.7-0.002x$. Solving $\frac{dP}{dx}=0$ gives the critical value $x=1350$.

Now $\frac{d^2P}{dx^2}=-0.002<0$, thus the unit of production for maximum profit is 1350.

(iii) Total profit, $P=4\times1350-(50+1.3\times1350+0.001\times1350\times1350)=1772.5$

Q. No. 5

(a) A farmer with 4000 meters of fencing wants to enclose a rectangular plot that borders on a river. If the farmer does not fence the side along the river, what is the largest area that can be enclosed?

(b) Prove that $\frac{\tan\theta+\sec\theta-1}{\tan\theta-\sec\theta+1}=\frac{1+\sin\theta}{\cos\theta}$

Solution: (a) Let x and y be the dimensions of the rectangular plot and let A be the area of the rectangle. Since the total length of the fence is 4000 meters, according to the question, we have

$$y=4000-2x \qquad (1)$$

and

$$A=xy=x(4000-2x)=4000x-2x^2 \qquad (2)$$

Calculate $\frac{dA}{dx} = 4000 - 4x$ and $\frac{d^2A}{dx^2} = -4 < 0$ so that A is maximum.

To find the dimension which will maximize A, we must have $\frac{dA}{dx} = 4000 - 4x = 0 \Rightarrow x = 1000$ so that $y = 4000 - 2000 = 2000.$

Hence, the maximum area is $A = 1000 \times 2000 = 20,00,000$ square meters.

(b) We can write

$$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{\sec\theta + \tan\theta - (\sec^2\theta - \tan^2\theta)}{1 - \sec\theta + \tan\theta} = \frac{\sec\theta + \tan\theta - (\sec\theta + \tan\theta)(\sec\theta - \tan\theta)}{1 - \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta + \tan\theta)(1 - \sec\theta + \tan\theta)}{1 - \sec\theta + \tan\theta} = \sec\theta + \tan\theta = \frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta} = \frac{1 + \sin\theta}{\cos\theta} \quad \text{(Proved).}$$

Q. No. 6

(a) If the equations $x^2 + px + q = 0$ and $x^2 + qx + p = 0$ have a common root, show that their other roots are the roots of the equation $x^2 + x + pq = 0.$

(b) The cost of producing 200 pens is Tk. 1000 and the cost of producing 400 pens is Tk.1500. (i) Find the linear relation between the cost y of producing x pens (ii) what number of pens must be produced and sold at Tk. 3 per pens, so that there is neither profit nor loss? (iii) What should be the selling price of a pen if 600 pens are produced and sold with a profit of Tk. 400?

Solution: **(a)** Given equations are $x^2 + px + q = 0 \quad (1)$

$$x^2 + qx + p = 0 \quad (2)$$

$$x^2 + x + pq = 0 \quad (3)$$

Let α be the common root of (1) and (2). So that $\alpha^2 + p\alpha + q = 0$ and $\alpha^2 + q\alpha + p = 0$.

We can write $(p-q)\alpha + q - p = 0 \Rightarrow (p-q)\alpha = p - q \Rightarrow \alpha = 1$

Now let β be the other root of (1). Then using formula, we get for eqn. (1),

$$1 + \beta = -p \Rightarrow \beta = -p - 1 \text{ and } 1 \times \beta = q \Rightarrow \beta = q$$

Therefore, we have the relation $\beta = -p - 1 \Rightarrow q = -p - 1 \Rightarrow p + q = -1 \quad (4)$

Again, let χ be the other root of (2). Then using formula, we get for eqn. (2),

$$1 + \gamma = -q \Rightarrow \gamma = -q - 1 \text{ and } 1 \times \gamma = p \Rightarrow \gamma = p \text{ and so } p + q = -1$$

The quadratic equation having roots p and q is $x^2 - (p+q)x + pq = 0 \Rightarrow x^2 + x + pq = 0$ since $p + q = -1$.

Hence, the other roots of (1) and (2) are the roots of (3).

(b) (i) Let $(x_1, y_1) = (200,1000)$ and $(x_2, y_2) = (400,1500)$.

So, the linear relation is

$$y - 1000 = \frac{1500 - 1000}{400 - 200}(x - 200) \Rightarrow 2(y - 1000) = 5(x - 200)$$

$$\Rightarrow 5x - 2y + 1000 = 0$$

Thus, $y = 2.5x + 500$ is the desired equation.

(ii) To produce 600 pens, revenue function, $R(x)=3x$ and cost function, $C(x)=2.5x+500$

We know that profit function = revenue function – cost function

$$\Rightarrow P(x)=3x-(2.5x+500)=0.5x-500$$

At the break-even point, we have neither profit nor loss and at the break-even point $P(x)=0$.

Therefore, $0.5x-500=0 \Rightarrow x=1000$

Hence, 1000 pens must be produced.

(iii) To produce 600 pens, total cost requires $y=2.5\times 600+500=2000$

Let *p* be the selling price.

We know that profit = revenue – cost $\Rightarrow 400 = p\times 600-2000 \Rightarrow p=4$

Thus, selling price is Tk. 4.

Q. No. 7

(a) Carryout the following: (i) $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$ (ii) $\int \frac{x}{\sqrt{2x+1}}dx$

(b) In a survey of 400 TV audiences, it was found that 150 regularly watch drama serial, 250 watch news program and 75 watch neither of these. How many watch the drama serial alone? How many watch news program alone?

Solution: (a) (i) We can write $\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \int \frac{e^x}{\sqrt{1-(e^x)^2}}dx$. Let $e^x = p$. Then $e^x dx = dp$.

Therefore, we get $\int \frac{e^x dx}{\sqrt{1-e^{2x}}} = \int \frac{dp}{\sqrt{1-p^2}} = \sin^{-1} p + c = \sin^{-1}(e^x)+c.$

(ii) To solve the given integral, let $2x+1=p^2$. Then $2dx = 2pdp \Rightarrow dx = pdp$.

Since $2x+1=p^2$, we get $x=\frac{1}{2}(p^2-1)$.

Therefore, $\int \frac{x}{\sqrt{2x+1}}dx = \frac{1}{2}\int \frac{p^2-1}{p}pdp = \frac{1}{2}\left(\frac{p^3}{3}-p\right)+c = \frac{1}{6}(2x+1)^{3/2}-\frac{1}{2}(2x+1)^{1/2}+c.$

(b) Let U be the universal set, D be the set of audiences who watch drama serials, and N be the set of audiences who watch news programs.

Let $n(D\cap N)$ be the number of audiences who watch both the programs, $n(D\cup N)'$ be the number of audiences who watch neither of the programs and let $n(D\cup N)$ be the number of audiences who watch at least one of the programs,

We know, $n(D\cup N)'=n(U)-n(D\cup N) \Rightarrow 75=400-n(D\cup N) \Rightarrow n(D\cup N)=325$

We also know, $n(D\cap N)=n(D)+n(N)-n(D\cup N) \Rightarrow n(D\cap N)=150+250-325=75$

Hence, we have the audiences who watch the drama serial alone is (150-75)=75 and the audiences who watch the news program alone is (250-75)=175.

CMA JUNE, 2017 EXAMINATION

PART – A: BUSINESS MATHEMATICS

Q. No. 1

(a) Let U = {1, 2, 3,......, 8, 9} be the universal set and let A = {1, 2, 3, 4} and B= {2, 4, 6, 8}. Write down the following sets:
(i) AUB, (ii) A∩B, (iii) A^c, (iv) $(A\cap B)^c$

(b) A machine depreciates at the rate of 15% of its value at the beginning of a year. The machine was purchased for Tk.1,00,000 and the scrap value realized when sold was Tk.50,250. Find out the number of years during which the machine was in use.

Solution: (a) Given $U=\{1,2,3,4,5,6,7,8,9\}$, $A=\{1,2,3,4\}$, $B=\{2,4,6,8\}$

(i) $A\cup B=\{1,2,3,4,6,8\}$

(ii) $A\cap B=\{2,4\}$

(iii) $A^c=U-A=\{5,6,7,8,9\}$

(iv) $(A\cap B)^c=U-(A\cap B)=\{1,3,5,6,7,8,9\}$

(b) Given $S=50,250,\ P=1,00,000,\ i=0.15$

We have $S=P(1-i)^n \Rightarrow 50,000=1,00,000(1-0.15)^n \Rightarrow -\ln 2=n\ln 0.85$

$$\Rightarrow -\ln 2=n\ln 0.85 \Rightarrow n=\frac{-\ln 2}{\ln 0.85}=4.27$$

The machine was in use in 4.27 years.

Q. No. 2

(a) From a group of 8 men and 5 women, five persons are to be selected to form a committee so that at least 2 men are there in the committee. In how many ways can it be done?

(b) Solve the following simultaneous equations graphically and algebraically:

$2x+3y=18$

$7x+5y=35$

Solution: (a) Firstly 2 men can be selected out of 8 men in ${}^8C_2$ ways or 28 ways. The remaining 3 are to be women and the number of ways in which they can be chosen out of 5 women are ${}^5C_3=10$.
So the desired number of ways = $28\times 10=280$.

(b) Given $2x+3y=18$ (1)

$7x+5y=35$ (2)

Multiplying eqn. (1) by 5 and eqn. (2) by 3, we get

$10x+15y=90$ (3)

$21x+15y=105$ (4)

Subtracting eqn. (3) from eqn. (4):

$$(21-10)x+(15-15)y=105-90 \Rightarrow x=\frac{15}{11}$$

Then substituting the value of x in (1):

$$2\left(\frac{15}{11}\right)+3y=18 \Rightarrow 3y=18-\frac{30}{11} \Rightarrow y=\frac{168}{11}$$

Hence, the solution is $(x, y)=\left(\frac{15}{11}, \frac{168}{11}\right)$.

Q. No. 3

(a) If α, β are the roots of $ax^2+2bx+c=0$, find the value of $(\alpha+\frac{1}{\alpha})^2+(\beta+\frac{1}{\beta})^2$ in terms of a, b and c.

(b) Find the value of $\sin A(1+\tan A)+\cos A(1+\cot A)$.

Solution: (a) Since α, β are the roots of $ax^2+2bx+c=0$, we can write $\alpha+\beta=-\frac{2b}{a}$ and $\alpha\beta=\frac{c}{a}$.

Therefore, $\left(\alpha+\frac{1}{\alpha}\right)^2+\left(\beta+\frac{1}{\beta}\right)^2=\alpha^2+\beta^2+2+\frac{1}{\alpha^2}+\frac{1}{\beta^2}+2$

$$=(\alpha+\beta)^2-2\alpha\beta+\frac{\alpha^2+\beta^2}{\alpha^2\beta^2}+4=(\alpha+\beta)^2-2\alpha\beta+\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}+4$$

$$=(\alpha+\beta)^2-2\alpha\beta+\frac{(\alpha+\beta)^2-2\alpha\beta}{(\alpha\beta)^2}+4=\left(-\frac{2b}{a}\right)^2-2\frac{c}{a}+\frac{\left(-\frac{2b}{a}\right)^2-2\frac{c}{a}}{\left(\frac{c}{a}\right)}+4$$

$$=\frac{4b^2}{a^2}-\frac{2c}{a}+\frac{\frac{4b^2}{a^2}-\frac{2c}{a}}{\frac{c^2}{a^2}}+4=\frac{4b^2-2ac}{a^2}+\frac{4b^2-2ac}{c^2}+4.$$

(b) We can write $\sin A(1+\tan A)+\cos A(1+\cot A)=\sin A\left(1+\frac{\sin A}{\cos A}\right)+\cos A\left(1+\frac{\cos A}{\sin A}\right)$

$$=\sin A+\cos A+\frac{\sin^2 A}{\cos A}+\frac{\cos^2 A}{\sin A}=\sin A+\cos A+\frac{\sin^3 A+\cos^3 A}{\sin A\cos A}$$

$$=\sin A+\cos A+\frac{(\sin A+\cos A)(\sin^2 A-\sin A\cos A+\cos^2 A)}{\sin A\cos A}$$

$$=\sin A+\cos A+\frac{(\sin A+\cos A)(1-\sin A\cos A)}{\sin A\cos A}$$

$$=\frac{\sin^2 A\cos A+\sin A\cos^2 A+\sin A+\cos A-\sin^2 A\cos A-\sin A\cos^2 A}{\sin A\cos A}$$

$$=\frac{\sin A+\cos A}{\sin A\cos A}=\frac{\sin A}{\sin A\cos A}+\frac{\cos A}{\sin A\cos A}=\sec A+\cos ecA.$$

Q. No. 4

(a) If Ø is positive and an acute angle, find the value of Ø satisfying cot^2 Ø +cosec Ø -5 = 0.

(b) Find $\frac{dy}{dx}$ of the following functions:

(i) y = (sinx) $^{\cos x}$ (ii) $x^2 - y^2 + 3x = 5y$

Solution: (a) Given eqn. is $\cot^2\phi + \cos ec^2\phi - 5 = 0 \Rightarrow \frac{\cos^2\phi}{\sin^2\phi} + \frac{1}{\sin^2\phi} - 5 = 0 \Rightarrow \cos^2\phi + 1 - 5\sin^2\phi = 0$

$\Rightarrow 1 - \sin^2\phi + 1 - 5\sin^2\phi = 0 \Rightarrow 2 - 6\sin^2\phi = 0 \Rightarrow 3\sin^2\phi = 1 \Rightarrow \sin^2\phi \Rightarrow \sin\phi = \pm\frac{1}{\sqrt{3}}$

If $\sin\phi = \frac{1}{\sqrt{3}} \Rightarrow \phi = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 35.26^0 \in [0, \pi/2]$

If $\sin\phi = -\frac{1}{\sqrt{3}} \Rightarrow \phi = \sin^{-1}\left(-\frac{1}{\sqrt{3}}\right) \approx -35.26^0 \notin [0, \pi/2]$

Hence, $\phi \approx 35.26^0$.

(b) (i) Given $y = (\sin x)^{\cos x}$.

First taking natural logarithm on both sides, we get

$$\ln y = \cos x \ln(\sin x)$$

Differentiating with respect to *x*, we get

$$\frac{1}{y}\frac{dy}{dx} = \cos x \times \frac{1}{\sin x}\cos x - \sin x \ln(\sin x)$$

$$\Rightarrow \frac{dy}{dx} = y[\cos x \cot x - \sin x \ln(\sin x)] = (\sin x)^{\cos x}[\cos x \cot x - \sin x \ln(\sin x)].$$

(b) (ii) Given equation is $x^2 - y^2 + 3x = 5y$

Differentiating with respect to *x*, we get

$$2x - 2y\frac{dy}{dx} + 3 = 5\frac{dy}{dx} \Rightarrow (5 + 2y)\frac{dy}{dx} = 2x + 3 \Rightarrow \frac{dy}{dx} = \frac{2x+3}{2y+5}.$$

Q. No. 5

(a) Find the profit maximizing output for the following revenue and cost functions: $R(x) = 2000x - 2x^2$; $C(x) = x^3 - 59x^2 + 1315x + 2000$.

(b) Solve $\log_9 x + 3\log_3 x = 14$.

Solution: (a) Given the revenue function, $R(x) = 2000x - 2x^2$ and the cost function, $C(x) = x^3 - 59x^2 + 1315x + 2000$.

We know that profit function, $P(x) = R(x) - C(x) = 2000x - 2x^2 - x^3 + 59x^2 - 1315x - 2000$

$\Rightarrow P(x) = -x^3 + 57x^2 + 685x - 2000$

To find the maximum profit, we must put $\frac{dP}{dx} = 0$, *i.e.*, $\frac{dP}{dx} = -3x^2 + 114x + 685 = 0$ which gives $x = 43.28$.

Now $\frac{d^2P}{dx^2} = -6x + 114 < 0$ at $x = 43.28$.
This means that maximum profit will occur at $x = 43.28$ units of output.

(b) Given equation is $\log_9 x + 3\log_3 x = 14$

We can write $\log_9 x + 3\log_3 x = 14 \Rightarrow \frac{\log x}{\log 3^2} + 3\frac{\log x}{\log 3} = 14 \Rightarrow \frac{\log x}{2\log 3} + \frac{3\log x}{\log 3} = 14$

$\Rightarrow \frac{\log x + 6\log x}{2\log 3} = 14 \Rightarrow 7\log x = 28\log 3$

$\Rightarrow \log x = 4\log 3 \Rightarrow \log x = \log 3^4 \Rightarrow x = 81.$

Q. No. 6

(a) Find the inverse of the matrix $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$.

(b) Carryout the following integrals: (i) $\int \left(\frac{e^x}{e^{2x}+1}\right) dx$ (ii) $\int x^2(2x^3+3)^5\, dx$

Solution: (a) Given $A = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 1 & 2 \\ 3 & 2 & 1 \end{pmatrix}$.

Now $|A| = 2(2-2) - 3(1-6) + 4(2-3) = 11,$
The cofactors of A are
$A_{11} = 0,\ A_{12} = 5,\ A_{13} = -1;\ A_{21} = 5,\ A_{22} = -10,\ A_{23} = 5;\ A_{31} = 2,\ A_{32} = 0,\ A_{33} = -1.$

Therefore, adjoint matrix, $adj\ A = \begin{bmatrix} 0 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix}$

Hence, $A^{-1} = \frac{1}{|A|} adj\ A = \frac{1}{11}\begin{bmatrix} 0 & 5 & 2 \\ 5 & -10 & 0 \\ -1 & 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 5/11 & 2/11 \\ 5/11 & -10/11 & 0 \\ -1/11 & 5/11 & -1/11 \end{bmatrix}$.

(b) (i) Given integral is $I = \int \left(\frac{e^x}{e^{2x}+1}\right) dx$

Let $e^x = p$. Then $e^x dx = dp$. Therefore, we get

$I = \int \left(\frac{e^x}{e^{2x}+1}\right) dx = \int \frac{dp}{p^2+1} = \tan^{-1} p + c = \tan^{-1}(e^x) + c.$

(b) (ii) Given integral is $I=\int x^2(2x^3+3)^5\,dx$

Let $2x^3+3=p$. Then $6x^2dx=dp \Rightarrow x^2dx=\frac{1}{6}dp$. Therefore, we get

$$I=\int x^2(2x^3+3)^5dx=\frac{1}{6}\int p^5dp=\frac{1}{6}\left(\frac{1}{6}p^6\right)+\text{c}=\frac{1}{36}(2x^3+3)^6+c.$$

Q. No. 7

(a) A man retires at the age of 59 years and his employer gives him a pension of Tk. 1,20,000 a year paid in half yearly installments for the rest of his life. Reckoning his expectation of life after retirement to be 15 years and that interest is at 4% p.a. payable half yearly, what single sum is equivalent to his pension?

(b) What is the present value of Tk. 1000 due in 2 years at 5% p.a. compound interest, according as the interest is paid half-yearly?

Solution: (a) Given $A=1,20,000,\ i=\frac{0.04}{2}=0.02,\ n=15\times 2=30$

We have $P=\frac{A}{i}\left[1-(1+\text{i})^{-\text{n}}\right]\Rightarrow P=\frac{1,20,000}{0.02}[1-(1.02)^{-30}]\Rightarrow P=26,87,574$

(b) Similar problems have been solved.

CMA DECEMBER, 2017 EXAMINATION

PART -A: BUSINESS MATHEMATICS

Q. No. 1

(a) If A = {1, 2, 3}, B = {2, 3, 4}, S = {1, 3, 4} and T = {2, 4, 5}, verify that (A×B) ∩ (S×T) = (A∩S) × (B∩T).

(b) The total cost y, for x units of a certain product consists of fixed cost and variable cost. It is known that the total cost is Tk.6000 for 500 units and Tk. 9000 for 1000 units.

(i) Find the linear relationship between x and y,

(ii) Find the slope of the line, what does it indicate?

(iii) Find the number of units that must be produced so that

(1) There is neither profit nor loss.

(2) There is a profit of Tk.1000.

(3) There is a loss of Tk.300; it being given that the selling price is Tk. 8 per unit.

Solution: (a) Given A = {1, 2, 3}, B = {2, 3, 4}, S = {1, 3, 4} and T = {2, 4, 5}.

Calculate $A\times B=\{(1,2),(1,3),(1,4),(2,2),(2,3),(2,4),(3,2),(3,3),(3,4)\}$

$S\times T=\{(1,2),(1,4),(1,5),(3,2),(3,4),(3,5),(4,2),(4,4),(4,5)\}$

Therefore, $(A\times B)\cap(S\times T)=\{(1,2),(1,4),(3,2),(3,4)\}$

Again, $A\cap S=\{1,3\}$ and $B\cap T=\{2,4\}$

Therefore, $(A\cap S)\times(B\cap T)=\{(1,2),(1,4),(3,2),(3,4)\}$

Hence, $(A\times B)\cap(S\times T)=(A\times B)\cap(S\times T)$ is verified.

(b) (i) Given $(x_1,y_1)=(500,6000)$ and $(x_2,y_2)=(1000,9000)$.

So, the linear relation is

$$y-6000=\frac{6000-9000}{500-1000}(x-500)\Rightarrow y=6x+3000$$

(ii) The slope of the line is $m=\dfrac{9000-6000}{1000-500}=6$

It indicates that if x increases by 1 unit then y increases by 6 units.

(iii) Given that selling price = Tk. 8

Therefore, revenue, $R(x)=8x$

(1) For neither profit nor loss, $R(x)=C(x)\Rightarrow 8x=6x+3000\Rightarrow x=1500$

(2) We have Profit = Revenue – Cost $\Rightarrow P(x)=R(x)-C(x)\Rightarrow 1000=8x-(6x+3000)\Rightarrow x=2000$

(3) We have Loss = Cost – Revenue $\Rightarrow 300=6x+3000-8x\Rightarrow x=1350$

Q. No. 2

(a) A sum of Tk. 50,000 invested for 4 years at 5% interest per annum compounded quarterly.
Find the effective rate of interest per annum.

(b) Mr. Abid can purchase a machine by paying Tk. 4,00,000 in cash now. He can also purchase the machine by 8 equal yearly installments to be paid at the beginning of each year. If the interest rate is 10% what should be amount of each installment?

Solution: (a) We know, the effective rate of interest, $i_{eff} = \left(1+\frac{i}{m}\right)^{m} - 1$

Given that $i = 5\%,\ m = 4$ and $n = 1$

Hence, the desired effective rate of interest is

$$i_{eff} = \left(1+\frac{0.05}{4}\right)^{1\times 4} - 1 = 5.09\%$$

(b) We know, present value $PV = \frac{AR}{R-1}\left(1-R^{-n}\right) \Rightarrow PV = \frac{A}{i}(1+i)\left[1-(1+i)^{-n}\right]$

$$\Rightarrow 400000 = \frac{A}{0.1}(1+.1)\left[1-.467\right]$$

$$\Rightarrow A = 68,224.46 \approx 68,224$$

Hence, Tk. 68,224 should be put for each installment.

Q. No. 3

(a) If α, β are the roots of the equation $(a+b+c)x^2 + (b+2c)x + c = 0$ form the equation whose roots are $\frac{\alpha}{\alpha+1}$ and $\frac{\beta}{\beta+1}$.

(b) There are 12 books of the same author, 5 copies of Business Mathematics, 4 copies of Statistics and 3 copies of Business Communications. In how many ways it is possible to make a selection by taking some or all of the books?

Solution: (a) If α, β are the roots of the equation $(a+b+c)x^2 + (b+2c)x + c = 0,$ then using formula, we can write $\alpha+\beta = -\frac{b+2c}{a+b+c}$ and $\alpha\beta = \frac{c}{a+b+c}$.

Then $$\frac{\alpha}{\alpha+1} + \frac{\beta}{\beta+1} = \frac{\alpha(\beta+1)+\beta(\alpha+1)}{(\alpha+1)(\beta+1)} = \frac{2\alpha\beta+(\alpha+\beta)}{\alpha\beta+(\alpha+\beta)+1} = \frac{2\frac{c}{a+b+c} - \frac{b+2c}{a+b+c}}{\frac{c}{a+b+c} - \frac{b+2c}{a+b+c} + 1}$$

$$= \frac{2c-(b+2c)}{c-(\mathrm{b}+2c)+a+b+c} = -\frac{b}{a}$$

and $$\frac{\alpha}{\alpha+1} \times \frac{\beta}{\beta+1} = \frac{\alpha\beta}{(\alpha+1)(\beta+1)} = \frac{\frac{c}{a+b+c}}{\frac{c}{a+b+c} - \frac{b+2c}{a+b+c} + 1} = \frac{c}{a}$$

Hence, the desired quadratic equation is given by

$$\left(\frac{\alpha}{\alpha+1}+\frac{\beta}{\beta+1}\right)x^2+\left(\frac{\alpha}{\alpha+1}\times\frac{\beta}{\beta+1}\right)x+c=0 \Rightarrow -\frac{b}{a}x^2+\frac{c}{a}x+c=0$$

$$\Rightarrow -bx^2+cx+ac=0 \Rightarrow bx^2-cx-ac=0$$

(b) Given that there are 12 books in total. There are 5 copies of Business Mathematics, 4 copies of Statistics and 3 copies of Business Communications. Therefore, selection can be made in

$$\frac{12!}{5!4!3!}=\frac{12\times11\times10\times9\times8\times7\times6\times5!}{5!\times4\times3\times2\times3\times2}=27,720 \text{ ways}$$

Q. No. 4

(a) Solve the following system using the inverse matrix:

$$3x + y + z = 1$$
$$2x + 2z = 0$$
$$5x + y + 2z = 2$$

(b) Solve the equation $\log_{10}(3x+2)-2\log_{10}x=1-\log_{10}(5x-3)$.

Solution: (a) The given system can be written in the matrix form as

$$AX = B, \text{ where } A=\begin{bmatrix}3&1&1\\2&0&2\\5&1&2\end{bmatrix}, X=\begin{pmatrix}x\\y\\z\end{pmatrix}, B=\begin{pmatrix}1\\0\\2\end{pmatrix}$$

Now $|A|=-2(2-1)-2(3-5)=2,$

The cofactors of A are

$A_{11}=-2,\ A_{12}=6,\ A_{13}=2;\ A_{21}=-1,\ A_{22}=1,\ A_{23}=2;\ A_{31}=2,\ A_{32}=-4,\ A_{33}=-2$

Therefore, adjoint matrix, $adj\,A=\begin{bmatrix}-2&-1&2\\6&1&-4\\2&2&-2\end{bmatrix}$

Thus $A^{-1}=\dfrac{adj\,A}{|A|}=\dfrac{1}{2}\begin{bmatrix}-2&-1&2\\6&1&-4\\2&2&-2\end{bmatrix}$

Since $AX = B,$ we have $X = A^{-1}B$.

i.e. $\begin{bmatrix}x\\y\\z\end{bmatrix}=\dfrac{1}{2}\begin{bmatrix}-2&-1&2\\6&1&-4\\2&2&-2\end{bmatrix}\begin{bmatrix}1\\0\\2\end{bmatrix}=\dfrac{1}{2}\begin{bmatrix}2\\-2\\-2\end{bmatrix}=\begin{bmatrix}1\\-1\\-1\end{bmatrix}$

Hence, the solution is $(x,y,z)=(1,-1,-1)$.

(b) Given equation is $\log_{10}(3x+2)-2\log_{10}x=1-\log_{10}(5x-3)$

We can write $\log_{10}(3x+2)-2\log_{10}x=1-\log_{10}(5x-3)$

$$\Rightarrow \log_{10}(3x+2) - 2\log_{10} x + \log_{10}(5x-3) = 1$$

$$\Rightarrow \log_{10} \frac{(3x+2)(5x-3)}{x^2} = \log_{10} 10$$

$$\Rightarrow \frac{(3x+2)(5x-3)}{x^2} = 10 \Rightarrow (3x+2)(5x-3) = 10x^2$$

$$\Rightarrow 15x^2 - 10x^2 - 9x + 10x - 6 = 0 \Rightarrow 5x^2 + x - 6 = 0$$

$$\Rightarrow 5x^2 + 6x - 5x - 6 = 0 \Rightarrow (5x+6)(x-1) = 0$$

$$\Rightarrow x = -6/5 \text{ or } x = 1$$

But if $x = -6/5$, then $3x+2 = 3\left(-\frac{6}{5}\right) + 2 = -\frac{18}{5} + 2 < 0$ and $5x - 3 = 5\left(-\frac{6}{5}\right) - 3 = -9 < 0$

Hence, the solution is $x = 1$.

Q. No. 5

(a) The profit made by a company that produces and sells x barrels of octane is $p(x)$ taka, where $p(x) = 100x - 0.01x^2$ -120,000.

(i) What should be x if profit is to be maximized?

(ii) What is the amount of the maximum profit?

(b) If $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$, prove that $\sin\theta + \cos\theta = \sqrt{2}\cos\theta$.

Solution: (a) Given the profit function, $p(x) = 100x - 0.01x^2 - 120,000$

To find the maximum profit, we must put $\frac{dp}{dx} = 0, i.e., \frac{dp}{dx} = 100 - 0.02x = 0 \Rightarrow x = 5000$

Now $\frac{d^2p}{dx^2} = -0.02 < 0$ so that profit is maximum at $x = 5000$ and

$$p(5000) = (100)(5000) - (0.01)(5000)^2 - 120,000 = 130,000$$

Hence, the maximum profit is Tk. 130,000.

(b) The given equation is $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

We can write $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$

$$\Rightarrow \cos\theta = \sqrt{2}\sin\theta + \sin\theta = (\sqrt{2}+1)\sin\theta$$

$$\Rightarrow \cos\theta = \frac{(\sqrt{2}+1)(\sqrt{2}-1)}{\sqrt{2}-1}\sin\theta = \frac{1}{\sqrt{2}-1}\sin\theta$$

$$\Rightarrow (\sqrt{2}-1)\cos\theta = \sin\theta$$

$$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta \text{ (Proved)}$$

Q. No. 6

(a) If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\ln x}{(\ln x + 1)^2}$.

(b) If $x^3 - 3x^2 y + 2y^2 - 5 = 0$, find $\frac{dy}{dx}$.

Solution: (a) Given $x^y = e^{x-y}$

Taking natural logarithm on both sides of this equation, we get

$$y\ln x = (x-y)\ln e \Rightarrow y\ln x = x-y$$

$$\Rightarrow y\times\frac{1}{x}+(\ln x)\frac{dy}{dx}=1-\frac{dy}{dx}$$

$$\Rightarrow (\ln x+1)\frac{dy}{dx}=1-\frac{y}{x}=\frac{x-y}{x}$$

Now $y\ln x = x-y \Rightarrow (\ln x+1)y = x \Rightarrow y=\dfrac{x}{\ln x+1}$ and so

Hence, we get $\dfrac{dy}{dx}=\dfrac{x-\dfrac{x}{\ln x+1}}{x(\ln x+1)}=\dfrac{x\ln x}{x(\ln x+1)^2}=\dfrac{\ln x}{(\ln x+1)^2}$. (Proved)

(b) Given eqn. is $x^3-3x^2y+2y^2-5=0 \Rightarrow 3x^2-6xy-3x^2\dfrac{dy}{dx}+4y\dfrac{dy}{dx}=0$

$$\Rightarrow (4y-3x^2)\frac{dy}{dx}=6xy-3x^2$$

$$\Rightarrow \frac{dy}{dx}=\frac{6xy-3x^2}{4y-3x^2}$$

Q. No. 7

(a) A company's loss is Tk. 121.50 in a certain year. Marginal Revenue and Marginal Cost of the company are MR = 30-6x and MC = -24+3x, respectively. Find profit function, break-even point and middle point between break-even.

(b) Integrate tan *x* with respect to *x*.

Solution: (a) Given marginal revenue, $MR = 30-6x$ and marginal cost, $MC=-24+3x$.

According to the question, cost – revenue = loss $\Rightarrow -24+3x-(30-6x)=121.5$

$$\Rightarrow 9x = 121.5+54=175.5 \Rightarrow x=19.5$$

Again, profit function, $P=30-6x-(-24+3x)=54-9x$

For break-even point, $P=0 \Rightarrow 54-9x=0 \Rightarrow x=6$

The break-even point is

Solution: (a) Revenue, $R=\int(30-6x)dx=30x-3x^2$

and cost, $C=\int(-24+3x)dx=-24x+\dfrac{3}{2}x^2$

We know, profit = Revenue – Cost $\Rightarrow P=30x-3x^2+24x-1.5x^2=54x-4.5x^2$

At break-even point, Revenue = Cost $\Rightarrow 30x-3x^2=-24x+1.5x^2 \Rightarrow 108x=9x^2 \Rightarrow x=12$

(b) We can write

$\int \tan x\,dx=\int\dfrac{\sin x}{\cos x}dx=-\ln|\cos x|+c=\ln|\sec x|+c$ since we have $\int\dfrac{f'(x)}{f(x)}dx=\ln|f(x)|+c.$