



THE INSTITUTE OF COST AND MANAGEMENT ACCOUNTANTS OF BANGLADESH
CMA JUNE, 2018 EXAMINATION
BUSINESS LEVEL
SUBJECT: GE 03. FUNDAMENTALS OF BUSINESS MATHEMATICS.

INDICATIVE SOLUTION

Q. No. 1

(a) Find the value of X in terms of a, b, c and Y, from: $Y = 2\sqrt{(abX \div c)}$

Solution: $Y = 2\sqrt{(abX \div c)}$

$$\Rightarrow Y^2 = 4(abX \div c)$$

$$\Rightarrow Y^2 = 4abX \div 4c$$

$$\Rightarrow 4abX = Y^2 \times 4c$$

$$\Rightarrow X = 4Y^2c \div 4ab$$

$$\Rightarrow X = Y^2c/ab. \text{ (Ans.)}$$

(b) Solve for x from: $-3x + 10 > 40$

Solution: $-3x + 10 > 40$

$$\Rightarrow -3x > 40 - 10$$

$$\Rightarrow -3x > 30$$

$$\Rightarrow x < \{30/(-3)\}$$

$$\Rightarrow x < -10. \text{ (Ans.)}$$

(c) A person pays no tax on the first Tk.2,50,000 of earnings and then 10% tax on the remainder of earnings. If s/he wishes to have Tk.6,10,000 net of tax earnings, what gross earnings does s/he need?

Solution: Here, taxable income net of tax is = Tk.6,10,000 – Tk.2,50,000
= Tk.3,60,000.

So, the taxable income corresponding to this Tk.3,60,000 is
=Tk.{3,60,000/(1- 0.10)}
=Tk.Tk.4,00,000

So, the person's gross earnings is = Tk.2,50,000 + Tk.4,00,000
=Tk.6,50,000. (Ans.)

(d) A small company produces specialized posters. The total cost is made up of three elements – materials, labor and administration as follows:

Materials	Tk.0.50 per poster
Labor	Tk.15 per hour
Administration	Tk.10 per 100 plus Tk.50

The set up time for printing takes two hours and the posters are run off at the rate of 300 per hour. If the number of posters produced is denoted by N, write down the formulae for the following in Tk.:

- (i) the total cost of materials,
- (ii) the time taken to produce the posters,
- (iii) the total labor cost of production,
- (iv) the administration cost.

Solution: Here, (i) Total cost of materials for N posters = 0.50N.
(ii) Time taken to produce N posters = 2 + N/300.
(iii) Total labor cost to produce N posters = 15 (2 + N/300).
(iv) Administrative cost = 10. (N/100) + 50 = 50 + N/10.

- (e) For statistical data, mention various types of the measure of dispersion. Which one is the best measure and why?

Solution: In statistical data the various measures of dispersion are:

- the range;
- the interquartile range;
- the quartile deviation;
- the mean absolute deviation;
- the standard deviation;
- the variance
- the coefficient of variation.

Among these measures, the standard deviation is the best measure because it tells how closely the values of the data set are clustered around the mean.

Q. No. 2.

- (a) State at least four characteristics of quality data.

Solution: Characteristics of good quality data are accuracy, completeness, reliability, relevance and timeliness. Accuracy means the information is correct in every detail. Completeness is the comprehensiveness of the information. Reliability is the ability of data to match with other trusted resources. Relevance means the one really needs the particular information. And timeliness is the ability of its use for real-time reporting.

- (b) A firm has the workforce with the following age structure:

Age-group (years)	25-30	30-35	35-40	40-45	45-50	50-55	55-60	60-65
Number of persons	5	10	13	28	21	12	8	3

Calculate the mean, median, mode, standard deviation and coefficient of variation.

Solution:

Age-group (Class)	Mid Point x	Frequency, f (no. of Prsn)	d=(x-42.5)/5	fd	fd ²	c.f.
25-30	27.5	5	-3	-15	45	5
30-35	32.5	10	-2	-20	40	15
35-40	37.5	13	-1	-13	13	28
40-45	42.5	28	0	0	0	56
45-50	47.5	21	+1	+21	21	77
50-55	52.5	12	+2	+24	48	89
55-60	57.5	8	+3	+24	72	97
60-65	62.5	3	+4	+12	48	100
		<u>ΣN=100</u>		<u>Σfd=+33</u>	<u>Σfd²=287</u>	

$$\text{Mean, } \bar{X} = A + \frac{\sum fd}{N} \times c = 42.5 + \frac{33}{100} \times 5 = \mathbf{44.15}$$

Here 50th class i.e. 40 – 45 is the median class.

$$\text{Median} = L + \frac{\frac{N}{2} - p.c.f}{f} \times c = 40 + \frac{\frac{100}{2} - 28}{28} \times 5 = \mathbf{43.93}$$

Here with the highest frequency 28, the class 40 – 45 is the modal class.

$$\text{So, Mode} = L + \frac{\Delta_1}{\Delta_1 + \Delta_2} \times c = 40 + \frac{28 - 13}{(28 - 13) + (28 - 21)} \times 5 = \mathbf{43.41}$$

$$\text{Standard deviation, } \sigma = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times c = \sqrt{\frac{287}{100} - \left(\frac{33}{100}\right)^2} \times 5 = \mathbf{7.8}$$

$$\text{Co-efficient of variation, C.V.} = \frac{\sigma}{\bar{X}} \times 100 = \frac{7.9}{44.15} \times 100 = \mathbf{17.89}$$

(c) Calculate Laspeyre's price index and Paasche's price index for the following data:

Meal Type	Meals for 2010		Meals for 2011	
	Price per meal in Tk.	Quantity	Price per meal in Tk.	Quantity
Fish & Chips	1800	20	2000	18
Pizza	1600	22	1800	17
Rice & Curry	2200	18	2400	20
Roast Lunch	2400	15	2600	11

Solution:

$$\text{Laspyere's Price Index in 2011} = \frac{\sum[\text{Price at Observation Period} \times \text{Base Quantity}]}{\sum[\text{Price at Base Period} \times \text{Base Quantity}]} \times 100$$

$$= \frac{(2000 \times 20) + (1800 \times 22) + (2400 \times 18) + (2600 \times 15)}{(1800 \times 20) + (1600 \times 22) + (2200 \times 18) + (2400 \times 15)} \times 100$$

$$= \frac{161800}{146800} \times 100$$

$$= 110.22$$

Paasche's Price Index in 2011

$$= \frac{\sum[\text{Price at Observation Period} \times \text{Observation Quantity}]}{\sum[\text{Price at Base Period} \times \text{Observation Quantity}]} \times 100$$

$$= \frac{(2000 \times 18) + (1800 \times 17) + (2400 \times 20) + (2600 \times 11)}{(1800 \times 18) + (1600 \times 17) + (2200 \times 20) + (2400 \times 11)} \times 100$$

$$= \frac{143200}{130000} \times 100$$

$$= 110.15$$

(d) A pie chart is used to display the following data:

- (i) Sales of product J 43
- (ii) Sales of product V 25
- (iii) Sales of product C 27

What angle in degrees on the pie chart would represent product C's share of sales?

Solution:

$$\begin{aligned} \text{Product C's share of sales in degrees on the pie chart is} \\ &= [360 \div (43 + 25 + 27)] \times 27 \text{ degrees} \\ &= [360 \div 95] \times 27 \text{ degrees} \\ &= 102.32 \text{ degrees.} \end{aligned}$$

Q. No. 3

- (a) An investor is considering three options, only one of which s/he can afford. All three have the same initial outlay, but there are different income patterns available from each. Investment A pays Tk.20,000 each year at the end of the next five years. Investment B pays Tk.10,000 at the end of the first year, Tk.15,000 at the end of the second year, and so on until the final payment of Tk.30,000 at the end of the fifth year. Investment C pays Tk.40,000 at the end of the first year, Tk.30,000 at the end of the second year, and Tk.20,000 at the end of the third. The investor estimates a constant rate of interest of 10% throughout the next 5 years. Which investment should s/he choose?
- (b) For project C above, if the initial outlay is Tk.70,000, then find the IRR for C. What is the profitability index of this project?
- (c) In project financial analysis, write down the accept-reject criterion for pay-back period, NPV, IRR and Profitability Index.

Solution:

Here to evaluate the acceptance of an option the PV of all the options will have to be determined.

$$(a) PV(A) = \frac{A}{r} \left[1 - \frac{1}{(1+r)^n} \right] = \frac{20000}{0.1} \times \left[1 - \frac{1}{(1+0.1)^5} \right] = 75,815.74$$

$$PV(B) = \left[\frac{Tk.10,000}{(1+0.10)^1} + \frac{Tk.15,000}{(1+0.10)^2} + \frac{Tk.20,000}{1.10^3} + \frac{Tk.25,000}{1.10^4} + \frac{Tk.30,000}{1.10^5} \right]$$

$$= Tk.72,216.88$$

$$PV(C) = \left[\frac{Tk.40,000}{(1+0.10)^1} + \frac{Tk.30,000}{(1+0.10)^2} + \frac{Tk.20,000}{1.10^3} \right] = Tk.76,183.32.$$

From the above analyses, the investor can choose option C as its PV is the largest.

- (b) With the initial outlay of Tk.70,000, the NPV @10% rate is $(76,183.32 - 70,000) = Tk.6,183.32$. If the discount rate is taken as 18%, then the NPV is

$$NPV(C) = -70,000 + \left[\frac{Tk.40,000}{(1+0.18)^1} + \frac{Tk.30,000}{(1+0.18)^2} + \frac{Tk.20,000}{(1+0.18)^3} \right] = -Tk.2,383.55.$$

$$\text{So, } IRR = 10\% + \frac{Tk.6,183.32}{Tk.6,183.32 - (-Tk.2,383.55)} \times (18\% - 10\%) = 15.77\%$$

$$\text{So, } PI = \frac{PV \text{ of CIF}}{PV \text{ of COF}} = \frac{Tk.76,183.32}{Tk.70,000} = 1.0883.$$

(C) Accept-reject Criteria for-

Pay-back period: If the pay-back period is less than some predetermined period, then the option is accepted, otherwise rejected.

NPV: If the NPV is positive, the project might be taken, if NPV is negative, the project is rejected and if NPV = 0 then it can be accepted or rejected.

IRR: If IRR is greater than the opportunity cost, then accepted. If IRR is less than the opportunity cost, it is rejected. But if IRR is equal to the opportunity cost then it can be accepted or rejected.

PI: If PI is greater than one the project is accepted. If PI is less than one, it is rejected. But if PI is equal to one then it can be accepted or rejected.

Q. No. 4

(a) Carefully study the following spread-sheet and answer the questions below:

	A	B	C	D
1	Unit selling price	Tk.65	Annual volume	10,000
2				
3	Seasonal Variation			
4	Quarter-1	-20%		
5	Quarter-2	-35%		
6	Quarter-3	10%		
7	Quarter-4	45%		
8				
9	Sales budget	Seasonal variation (units)	Quarterly volume	Quarterly turnover
10	Quarter-1			
11	Quarter-2			
12	Quarter-3			
13	Quarter-4			

- (i) The cell B10 shows the seasonal variation in units for quarter-1. Write a suitable formula for this cell so that it can be copied for cells B11, B12 and B13.
- (ii) The cell C10 shows the sales volume in units for quarter-1. Write a suitable formula for this cell so that it can be copied for cells C11, C12 and C13.
- (iii) The cell D10 shows the turnover in quarter-1. Write a suitable formula for this cell so that it can be copied for cells D11, D12 and D13.

Solution: (a)

- (i) $B10=(\$D\$1/4)*B4$
- (ii) $C10=(\$D\$1/4)*(1+B4)$
- (iii) $D10=C10*\$B\1

(b) The table below shows the demand for a particular brand of razor in a shop for each of the last nine months.

Month	1	2	3	4	5	6	7	8	9
Demand	10	12	13	17	15	19	20	21	20

- (i) Calculate a three month moving average for months three to nine. What would be your forecast for the demand in month ten?
- (ii) Apply exponential smoothing with a smoothing constant of 0.3 to derive a forecast for the demand in month ten.
- (iii) Which of the two forecasts for month ten do you prefer and why?

Solution: (i)

Period	Month	Demand (D)	3-month total (T)	3-month average (A)	Epo. Smooth. $\alpha = 0.3$	Remarks
1	1	10	-		10	The formula is: $A_{t+1}=(D_t+ D_{t-1}+ D_{t-2})/3$
2	2	12	-		10	
3	3	13	-		10.6	
4	4	17	35	11.67	11.32	
5	5	15	42	14.00	13.024	
6	6	19	45	15.00	13.6168	
7	7	20	51	17.00	15.23176	
8	8	21	54	18.00	21.430472	
9	9	20	60	20.00	20.8709	
10	10		61	20.33	19.7387	

Hence, the month 10 demand would be 20.33.

(ii) Smoothing constant $\alpha = 0.3$.

Now from $F_{t+1} = F_t + \alpha (D_t - F_t)$

For first period, let $F_1 = D_1 = 10$

So, $F_2 = F_1 + \alpha (D_1 - F_1) = 10 + 0.3 (10 - 10) = 10$

$F_3 = F_2 + \alpha (D_2 - F_2) = 10 + 0.3 (12 - 10) = 10.6$

$F_4 = F_3 + \alpha (D_3 - F_3) = 10.6 + 0.3 (13 - 10.6) = 11.32$

Thus calculating we can get from the above table, the month 10 demand is 19.7387≈20 (rounded).

(iii) From the calculation, if rounded the demand forecasted for month 10 are same. Taking the full figure the later forecast is preferable because of the trend of period 7, 8 and 9.

(c) Carefully study the following spread-sheet and answer the questions below:

	A	B	C	D	E
1					
2					
3					
4		5	6	8	=B4+C4*D4
5		3	1	2	=(B5+C5)*2
6		9	7	2	=B6-C6/2
7		4	5	10	=(B7+C7)/D7
8		5	7	8	=B8+C8*D8-2
9		6	6		=(B9-1)*(C9+4)
10					
11					
12					
13					
14		9	7	-2	=B14-C14*D14
15		5	8	-6	=(B15-C15)^2
16		12	8	-4	=(B16^2)+(C16)
17		4	16	-2	=(B16/C17)*D17^2
18		17	8		=(B18+C18)/0.5
19		7	20	6	=C19/(B19+C19)

From the above spreadsheet find the values of E4, E5, E6, E7, E8, E9, E14, E15, E16, E17, E18, and E19.

Solution:

$$E4=B4+C4*D4=5+6*8=5+48=53$$

$$E5=(B5+C5)*2=(3+1)*2=8$$

$$E6=B6-C6/2=9-7/2=9-3.5=5.5$$

$$E7=(B7+C7)/D7=(4+5)/10=9/10=0.9$$

$$E8=B8+C8*D8-2=5+7*8-2=5+56-2=59$$

$$E9=(B9-1)*(C9+4)=(6-1)*(6+4)=5*10=50$$

$$E14=B14-C14*D14=9-7*(-2)=9+14=23$$

$$E15=(B15-C15)^2=(5-8)^2=9$$

$$E16=(B16^2)+(C16)=12^2+8=144+8=152$$

$$E17=(B16/C17)*D17^2=(12/16)*(-2)^2=(12/16)*4=3$$

$$E18=(B18+C18)/0.5=(17+8)*0.5=25*0.5=12.5$$

$$E19=C19/(B19+C19)=20/(7+20)=20/27=0.74074.$$

(d) Mrs. Sifat wants to plan a trip to Hawaii with her husband on their 10th wedding anniversary in two years. She anticipates that the all-inclusive trip will cost \$9500 for both of them and wants to start saving. How much should she deposit at the end of each month into her account that pays 12% compounded monthly to pay for the trip in two years?

Solution: Here, the future value (FV) is \$9500, Period n = 24 months, rate of interest r=12%.

We know,
$$FV = \frac{A}{\frac{r}{12}} \left[\left(1 + \frac{r}{12} \right)^{12n} - 1 \right]$$

$$\Rightarrow 9500 = \frac{A}{\frac{.12}{12}} \left[\left(1 + \frac{.12}{12} \right)^{12 \times 2} - 1 \right]$$

$$\Rightarrow A = \$352.20$$

So, the monthly deposit would be \$352.20.

Q. No. 5

(a) A pet food company has developed a low-fat dog food designed to make overweight dogs loss weight. In field tests on 1,000 pet dogs, some dogs ate the new food and some a normal dog food. The results of these tests were as follows:

	Given low-fat food	Given normal food
Lost weight	200	250
No weight loss	300	250

Find:

- (i) The probability that a dog has lost weight.
- (ii) The conditional probability that a dog has lost weight, given that it received low-fat food.
- (iii) The conditional probability that a dog was given the normal food, given that it lost weight.

Solution: (a) Here,

	Given low-fat food	Given normal food	Total
Lost weight	200	250	450
No weight loss	300	250	550
	500	500	1000

- (i) $P(\text{a dog lost weight}) = 450/1000=0.45$
- (ii) $P(\text{lost weight} | \text{low-fat food}) = 200/500 = 0.40$
- (iii) $P(\text{normal food} | \text{lost weight}) = 250/450 = 0.56.$

- (b) A salesman is known to sell a product in 3 out of 5 attempts while another salesman is 2 out of 5 attempts. Find the probability that
- (i) No sale will be effected when they both try to sell the product and
 - (ii) Either of them will succeed in selling the product.

Solution: (b) Here, $P(\text{Sale by first person}) = 3/5$; so $P(\text{No sale by first person}) = 1 - 3/5 = 2/5$.

Similarly, $P(\text{Sale by second person}) = 2/5$; so $P(\text{No sale by second person}) = 1 - 2/5 = 3/5$.

- (i) $P(\text{No sale by both}) = P(\text{No sale by first person}) \times P(\text{No sale by second person})$
 $= (2/5) \times (3/5)$
 $= 6/25$
 $= 0.24.$
- (ii) $P(\text{Either of them succeed})$
 $= P(\text{Sale by first person}) \times P(\text{No sale by second person}) + P(\text{No sale by first person}) \times P(\text{Sale by second person})$
 $= (3/5) \times (3/5) + (2/5) \times (2/5)$
 $= 9/25 + 4/25$
 $= 13/25$
 $= 0.52.$

- (c) A West Coast Publishing Company keeps accurate records of its monthly expenditure for advertising and its total monthly sales. For the first ten months of 2017, the records showed the following (note that units are in dollars):

Advertising (in thousands)	43	44	36	38	47	40	41	54	37	46
Sales (in millions)	74	76	60	68	79	70	71	94	65	78

- (i) Find the least-squares prediction equation appropriate for the data.
- (ii) If the company plans to spend \$80,000 for advertising next month, what is their predicted sale? (Assume that all other factors can be neglected).

Solution: (c) (i) Here, if the equation be of the form $y = a + bx$, then the following calculations can be made to get the values of a and b:

Advertising ('000), x	Sales (million), y	x^2	xy
43	74	1849	3182
44	76	1936	3344
36	60	1296	2160
38	68	1444	2584
47	79	2209	3713
40	70	1600	2800
41	71	1681	2911
54	94	2916	5076
37	65	1369	2405
<u>46</u>	<u>78</u>	<u>2116</u>	<u>3588</u>
$\Sigma x=426$	$\Sigma y=735$	$\Sigma x^2=18416$	$\Sigma xy=31763$

$$b = \frac{n \sum xy - (\sum x)(\sum y)}{n \sum x^2 - (\sum x)^2} = \frac{10 \times 31763 - 426 \times 735}{10 \times 18416 - (426)^2} = \frac{4520}{2684} = 1.6841$$

$$\bar{X} = \frac{426}{10} = 42.6 \quad \text{and} \quad \bar{Y} = \frac{736}{10} = 73.6$$

$$a = \bar{Y} - b.\bar{X} = 73.6 - 1.6841 \times 42.6 = 1.86$$

So, the required equation is, **$y = 1.86 + 1.68x$** .

(ii) If the advertisement expenditure is \$80,000, then the sale will be = $1.86 + 1.68 \times 80 = 136.26$ million.

Q. No. 6

(a) Solve the following simultaneous equations graphically and algebraically:

$$\begin{aligned} 4x + 3y &= 24 \\ 7x + 5y &= 35 \end{aligned}$$

Solution: $4x + 3y = 24$ ----- (i)
 $7x + 5y = 35$ ----- (ii)

From equation (i) we get, $3y = 24 - 4x$
 $\Rightarrow y = (24 - 4x)/3$

Putting this value of y in equation (ii),

$$\begin{aligned} 7x + 5[(24 - 4x)/3] &= 35 \\ \Rightarrow 7x + (120 - 20x)/3 &= 35 \\ \Rightarrow 21x + 120 - 20x &= 105 \quad [\text{multiplying both sides by 3}] \\ \Rightarrow x + 120 &= 105 \\ \Rightarrow x &= 105 - 120 \\ \Rightarrow x &= -15 \end{aligned}$$

So, $y = (24 - 4x)/3 = \{24 - 4(-15)\}/3 = (24 + 60)/3 = 28$
 That is $x = -15$ and $y = 28$ (solved).

(b) You have been supplied with the following data regarding the grades of candidates in an interview presentation and their written exam scores:

Candidate	Grade awarded	Exam score
Mr. A	A	70
Mr. B	B	76
Mr. C	A	58
Mr. D	C	88
Mr. E	D	81

What is the Spearman's correlation coefficient?

Solution:

Ranking the interview grades with best is 1 and also the exam scores:

Candidate	Rank of Grade	Rank of Score	d	d ²
A	1.5	4	-2.5	6.25
B	3	3	0	0.00
C	1.5	5	-3.5	12.25
D	4	1	3	9.00
E	5	2	3	9.00
Total:				<u>36.50</u>

Hence, the Spearman's correlation coefficient is

$$R = 1 - \frac{6 \sum d^2}{n(n^2 - 1)} = 1 - \frac{6 \times 36.50}{5(25 - 1)} = -0.825$$

(c) What is the difference between "risk" and "uncertainty"?

Ans.: Difference between risk and uncertainty:

Risk	Uncertainty
The probability of winning or losing something worthy is known as risk.	Uncertainty implies a situation where the future events are not known.
It can be measured.	It cannot be measured.
Chances of outcomes are known.	Chances of outcome is unknown.
Controllable.	Uncontrollable.
Minimization is possible.	Minimization is not possible.
Probability can be assigned.	Probability cannot be assigned.

(d) Write a short note on annuity.

Ans.: Annuity is a series of receipts or payments of a fixed amount for a specified number of periods/years. It is a pattern of cash flows that are equal in each year. The interval is generally a year, but it may be of six months or a quarter or a month. There are mainly two types of annuity problems:

1. Compound value (Terminal Value- TV) of an annuity: To get a fixed amount after a specified period what installment should be deposited per year or installment, if there is a fixed rate of interest.
2. Present value (PV) of an annuity: To get a series of equal amount per period by depositing a fixed amount at the beginning, if there is a specific rate of interest.

(e) In the analysis of inter-relationship of variables, the obtained coefficient of determination is 0.49. What do you mean by this figure?

Ans.: The coefficient of determination is a measurement used to explain how much variability of one factor can be caused by its relationship to another related factor. A value of 0.49 suggests that 49% of the dependent variable is predicted by the independent variable.

(f) In order of mathematical operations, what is meant by BEDMAS? How the sequence of the letters can be explained?

Ans.: In math, there is an agreed-upon set of procedures for the order in which operations are performed. When follow the correct order, the answer will be correct. We should work from left to right as used the BEDMAS order of operations. Each letter stands for:

- B - Brackets
- E - Exponents
- D - Division
- M - Multiplication
- A - Addition
- S - Subtraction.

(g) In the model of time series, $Y = T \times S \times C \times R$, which letter represents the seasonal variation? How this seasonal variation affects business decision? Explain shortly.

Ans.: The seasonal variation/component is represented by *S*. The *seasonal* component accounts for the regular variations that certain variables show at various times of the year. Business decision is seriously affected by seasonal variation because the demand of a product changes due to the variation. Umbrella is mostly demanded during the rainy season and least demanded in the winter in Bangladesh. So, production decision is affected with the intensity of seasonal variation.

= THE END =