CMA DECEMBER, 2018 EXAMINATION

FOUNDATION LEVEL

SUBJECT: 003. QUANTITATIVE TECHNIQUES

Time: Three hours | Full Marks: 100

- Answer **TEN** questions. FIVE questions from each part.
- Answer must be brief, relevant, neat and clean.
- Use fresh sheet for answering each question.

**PART – A: BUSINESS MATHEMATICS (Solution)**

**Q. No. 1** **[Marks: (5+5) = 10]**

(a) Dhaka city has a total population 2,00,00,000. Out of it 1,20,00,000 are service holder and 60,00,000 are businessmen while 10,00,000 are in both positions. Indicate how many people are neither service holders nor businessmen.

**Solution:**

Number of Population n(P) = 2,00,00,000

Number of Service holder n(S) = 1,20,00,000

Number of Businessmen n(B) = 60,00,000

Number of both $n(S \cap B)$ = 10,00,000

We know that, $n(S \cup B) = n(S) + n(B) - n(S \cap B)$

$n(S \cup B)$ = 1,20,00,000 + 60,00,000 – 10,00,000

$n(S \cup B)$ = 1,70,00,000

Hence, the number of neither Service holder nor Businessmen

$n(S \cup B)' = n(P) - n(S \cup B)$

= 2,00,00,000 – 1,70,00,000

= 30,00,000

(b) Each student in a class of 40 plays at least one indoor game chess, carrom and scrabble. 18 play chess, 20 play scrabble and 27 play carrom. 7 play chess and scrabble, 12 play scrabble and carrom and 4 play chess, carrom and scrabble.

Find the number of students who play (i) chess and carrom. (ii) Chess, carrom but not scrabble.

**Solution:**

Number of player n (P) = 40

Number of chess player n(C) = 18

Number of scrabble player n(S) = 20

Number of carrom player n (Cr) = 27

Hence, n (C ∩ S) = 7

N (S ∩ Cr) = 12

N (C ∩ Cr ∩ S) = 4

N (C ∩ Cr) =?

N (C ∩ Cr ∩S/) =

(i) We know that

n(C U S U Cr) = n(C) + n(S) + n(Cr) - n(C ∩ S) - n(S ∩ Cr) - n(C ∩ Cr) + n(C ∩ S ∩ Cr)

40 = 18 + 20 + 27 - 7 – 12- n(C ∩ Cr) +4

n(C ∩ Cr) = 50-40 = 10

The number of student 10 who play chees and carom.

(ii) N (C ∩ Cr ∩S/) = n(C ∩ Cr)- n(C ∩ S ∩ Cr) = 10-4 = 6

The number of student 6 who play chees, carom but not scrabble.

**Q. No. 2** **[Marks: (5+5) = 10]**

(a) If α, β are the roots of the equation $7x^2 - 5x - 3 = 0$. Form the equation which have the roots $\frac{1}{\alpha} + \frac{3}{\beta}, \frac{3}{\alpha} + \frac{1}{\beta}$.

**Solution:**

Given that $7x^2 - 5x - 3 = 0$ has two roots $\alpha \ and \ \beta$

Hence $\alpha + \beta = \frac{5}{7} \ and \ \alpha\beta = -\frac{3}{7}$

Now the summation of $\frac{1}{\alpha} + \frac{3}{\beta} \ and \ \frac{3}{\alpha} + \frac{1}{\beta}$ is

$$\frac{1}{\alpha} + \frac{3}{\beta} + \frac{3}{\alpha} + \frac{1}{\beta} = 4\left(\frac{\alpha+\beta}{\alpha\beta}\right) = 4 \times \frac{5}{7} \times \left(-\frac{7}{3}\right) = -\frac{20}{3}$$

And the product of $\frac{1}{\alpha} + \frac{3}{\beta} \ and \ \frac{3}{\alpha} + \frac{1}{\beta}$ is

$$\left(\frac{1}{\alpha} + \frac{3}{\beta}\right)\left(\frac{3}{\alpha} + \frac{1}{\beta}\right) = \frac{3}{\alpha^2} + \frac{1}{\alpha\beta} + \frac{9}{\alpha\beta} + \frac{3}{\beta^2} = \frac{3\alpha^2+\alpha\beta+9\alpha\beta+3\beta^2}{\alpha^2\beta^2}$$

$$= \frac{3(\alpha^2+\beta^2)+10\alpha\beta}{(\alpha\beta)^2} = \frac{3\{(\alpha+\beta)^2-2\alpha\beta\}+10\alpha\beta}{(\alpha\beta)^2}$$

$$= \frac{3(\alpha+\beta)^2+4\alpha\beta}{(\alpha\beta)^2} = \frac{3\times\frac{25}{49}+4\left(\frac{-3}{7}\right)}{\frac{9}{49}} = \frac{\frac{75-84}{49}}{\frac{9}{49}} = -\frac{9}{49} \times \frac{49}{9} = -1$$

Hence the required equation is $x^2 - (-\frac{20}{3})x + (-1) = 0$

$$x^2 + \frac{20}{3}x - 1 = 0$$

$$3x^2 + 20x - 3 = 0$$

(b) How many arrangements can be made with the letters of the word MATHEMATICS and in how many of them vowels occur together?

**Solution:**

The word MATHEMATICS has 11 letters in which the repeating letters are 2 Ms, 2 As and 2 Ts and the rest all different.

Hence it can be arranged in $\frac{11!}{2!2!2!} = 4989600$ different ways

2nd part,

There are 4 vowels in which repeated vowels is 2 As. If all vowels are grouped together. So there are effectively only 8 letters left. Thus we would write as $\frac{8!}{2!2!}$ and for the vowels $\frac{4!}{2!}$.

It can be arranged when all vowels are together $\frac{8!}{2!2!} \times \frac{4!}{2!} = 120960$ different ways.

**Q. No. 3** **[Marks: (4+6) =10]**

(a) Find the present value payment of Tk. 2000, 15 years from now, assuming that we discount at a rate of 5% per year compounded quarterly.

**Solution:**

We use the following formula to calculate the present value

$$P = \frac{F}{(1+i)^n} = F \times (1+i)^{-n}$$

Here the amount 15 years hence is F = Tk. 2000.

With quarterly compounding n = (4 periods per year) x (15 years) = 60 periods

$$i = \frac{0.05}{4} = 0.0125$$

Therefore $P = F \times (1+i)^{-n} = 2000 \times (1 + 0.013)^{-60} = 921.43$

The present value = Tk. 921.43.

(b) If x= tan ϴ + sec ϴ, show that Sin ϴ = $(x^2-1)/(x^2+1$

**Solution:**

Given that,

$x = tan\theta + sec\theta$

Or, $x = \frac{sin\theta}{cos\theta} + \frac{1}{cos\theta}$

Or, $x = \frac{sin\theta+1}{cos\theta}$

Or, $x^2 = \frac{(1+sin\theta)^2}{cos^2\theta} = \frac{(1+sin\theta)^2}{1-sin^2\theta} = \frac{1+sin\theta}{1-sin\theta}$

Or, $\frac{x^2-1}{x^2+1} = \frac{1+sin\theta-1+sin\theta}{1+sin\theta+1-sin\theta} = \frac{2\,sin\theta}{2} = sin\theta$ (Proved)

**Q. No. 4** **[Marks: (4+6) = 10]**

(a) Solve: 42x +33y - 117 = 0 and 48x + 27y – 123 = 0

**Solution:**

Given the equations

$$42x + 33y - 117 = 0$$

$$48x + 27y - 123 = 0$$

Using the cross multiplications formula, we have

$$\frac{x}{33\times(-123)-27\times(-117)} = \frac{y}{48\times(-117)-42\times(-123)} = \frac{1}{42\times27-48\times33}$$

Or, $\frac{x}{-900} = \frac{y}{-450} = \frac{1}{-450}$

Or, $\frac{x}{2} = \frac{y}{1} = \frac{1}{1}$

Hence the solution (x, y) = (2, 1)

(b) A Machine depreciates at the rate of 10% of its value at the beginning of a year. The machine was purchased for Tk. 25,000 and it was sold eventually as waste metal for Tk. 4,500. Obtain the number of year during which the machine was in use.

**Solution:**

We have $A = P(1-i)^n$

Here P = 25,000; A = 4500 and i = 10/100, n =?

So, $4500 = 25000\left(1 - \frac{10}{100}\right)^n \Rightarrow \left(\frac{9}{10}\right)^n = \frac{45}{250} \Rightarrow \left(\frac{9}{10}\right)^n = \frac{9}{50} \Rightarrow n(log9 - log10) = log9 - log50$

$$n(log9 - log10) = log9 - log50 \Rightarrow n(0.9542 - 1) = 0.9542 - 1.6990$$

$$n(0.9542 - 1) = 0.9542 - 1.6990 \Rightarrow n = \frac{-0.7448}{-0.0458} = 16\ years\ (approx)$$

**Q. No. 5** **[Marks: (6+4) = 10]**

(a) A manufacturing unit produces three types of products A, B and C which it sells in two markets. Annual sales volume is as follows:

| Markets | Products | | |
|---|---|---|---|
| | A | B | C |
| I | 4,000 | 3,000 | 2,000 |
| II | 3,000 | 2,000 | 1,000 |

(i) If the unit cost of the above three commodities are Tk. 2.00, Tk. 1.50 and Tk. 1.00 respectively and

(ii) If unit sale prices of A, B and C are Tk. 2.50, Tk. 2.00 and Tk. 1.50 respectively. Then find the total revenue in each market and total profit of both the markets with the help of matrix algebra.

**Solution:**

Let A be the matrix of the three types of products in both markets. C be the row matrix of the cost of requirements per unit; P be the row matrix of the selling price per unit of the above product A, B, C.

$A = \begin{pmatrix} 4000 & 3000 \\ 3000 & 2000 \\ 2000 & 1000 \end{pmatrix}$; $C = \begin{pmatrix} 2 & 1.5 & 1 \end{pmatrix}$; and $P = \begin{pmatrix} 2.5 & 2 & 1.5 \end{pmatrix}$

The total revenue in each market is obtained by the following way

$$PA = \begin{pmatrix} 2.5 & 2 & 1.5 \end{pmatrix} \times \begin{pmatrix} 4000 & 3000 \\ 3000 & 2000 \\ 2000 & 1000 \end{pmatrix} = \begin{pmatrix} 19000 & 13000 \end{pmatrix}$$

Total revenue in market I = Tk. 19000

Total revenue in market II = Tk. 13000

Therefore, total revenue in both markets = Tk. 32000

The total cost in producing the three types of product in both markets is obtained by the following way:

$$CA = \begin{pmatrix} 2 & 1.5 & 1 \end{pmatrix} \times \begin{pmatrix} 4000 & 3000 \\ 3000 & 2000 \\ 2000 & 1000 \end{pmatrix} = \begin{pmatrix} 14500 & 10000 \end{pmatrix}$$

Total cost = Tk. 24500

We know that, Profit = TR-TC = 32000-24500 = 7500

Therefore, the total profit of producing the above products is Tk. 7500.

(b) Differentiate with respect to x:

(i) a sin x

(ii) x sin x

**Solution:**

(i) $\frac{d}{dx}(a\ sinx) = a\ \frac{d}{dx}(sinx) + sinx\ \frac{d}{dx}(a) = a\ cosx + 0 = a\ cosx$

(ii) $\frac{d}{dx}(x\ sinx) = x\ \frac{d}{dx}(sinx) + sinx\ \frac{d}{dx}(x) = x\ cosx + sinx$

**Q. No. 6** **[Marks: (5+5) = 10]**

(a) A company has examined its cost structure and revenue structure and has determined that C the total cost, R total revenue and X the number of unit's products are related as: C = x + 2x$^2$ $-\frac{x^3}{3}$ and R = 5x. Show that the firm has no maximum or minimum profit.

**Solution:**

Here the profit function is

$$P(x) = R(x) - C(x) = 5x - x - 2x^2 + \frac{x^3}{3} = \frac{x^3}{3} - 2x^2 + 4x$$

$$\frac{dP}{dx} = x^2 - 4x + 4$$

For critical points $\frac{dP}{dx} = 0 \Rightarrow x^2 - 4x + 4 = 0 \Rightarrow x = 2$

$$\frac{d^2P}{dx^2} = 2x - 4 \Rightarrow \frac{d^2P}{dx^2} = 0 \; at \; x = 2$$

Thus the firm has no maximum or minimum profit.

(b) (i) $\int_3^5 \sqrt{x-3}\, dx$

(ii) $\int_1^2 (2x^3 - 1)\, 6x^2 dx$

**Solution:**

(i) $\int_3^5 \sqrt{x-3}\, dx$

Let, $x - 3 = y^2$

$x = y^2 + 3$

$dx = 2ydy$

When $x = 3, y = 0 \; and \; x = 5, y = \sqrt{2}$

Hence $\int_3^5 \sqrt{x-3}\, dx = \int_0^{\sqrt{2}} y\, 2y\, dy = 2\int_0^{\sqrt{2}} y^2\, dy = 2\left[\frac{y^3}{3}\right]_0^{\sqrt{2}} = 2 \times \frac{2\sqrt{2}}{3} = \frac{4\sqrt{2}}{3}$

(ii) $\int_1^2 (2x^3 - 1) 6x^2 dx = \int_1^2 (12x^5 - 6x^2) dx = [2x^6 - 2x^3]_1^2$

$= (2 \times 2^6 - 2 \times 2^3 - 2 \times 1^6 + 2 \times 1^3) = 112$

**Q. No. 7** **[Marks: (5+5) = 10]**

(a) Find the equation to the straight line which passes through the point of inter section of the straight line$2x + 3y + 4 = 0$ and $3x + 4y - 5 = 0$ and is perpendicular to the straight line $6x - 7y + 8 = 0$.

**Solution:** Given the equations

$2x + 3y + 4 = 0$ ............(1)

$3x + 4y - 5 = 0$ ............(2)

$6x - 7y + 8 = 0$ ............(3)

The intersecting point of (1) and (2) is

$\frac{x}{-13-16} = \frac{y}{12+10} = \frac{1}{8-9}$

$\frac{x}{-31} = \frac{y}{22} = -1$

$x = 31, y = -22$

Hence (x, y) = (31, -22)

Equation of straight line perpendicular to (3) is

$7x + 6y + k = 0$ .........(4)

Which passes through (31, -22)

$$7 \times 31 + 6(-22) + k = 0$$

$$k = -85$$

Hence from (4) we get $7x + 6y - 85 = 0$

(b) Solve:$4^{1+x} + 4^{1-x} = 10$

**Solution:** Given that,

$4^{1+x} + 4^{1-x} = 10$, or $4^1 4^x + \frac{4}{4^x} = 10$

Let, $4^x = y$, then $4y + \frac{4}{y} = 10$, or $4y^2 - 10y + 4 = 0$, or $(y-2)(4y-2) = 0$,

or $y = 2$, $y = \frac{1}{2}$

$4^x = y \Rightarrow 4^x = 2 \Rightarrow 4^x = 4^{\frac{1}{2}} \Rightarrow x = \frac{1}{2}$

or$4^x = y \Rightarrow 4^x = \frac{1}{2} \Rightarrow 4^x = 4^{-\frac{1}{2}} \Rightarrow x = \frac{-1}{2}$

or $x = \frac{1}{2}$ , or $x = -\frac{1}{2}$

**= THE END =**